National Council on Teacher Quality

Research Summary: Elementary Math

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Strong early math skills lay the foundation for success across all subjects. Students' early proficiency in math is linked to long-term achievement in reading, science, and overall academic performance, as well as lower rates of grade retention through eighth grade.[1] The benefits extend beyond the classroom: Improved math performance is associated with higher lifetime earnings, particularly for historically underrepresented students.[2] Advanced math coursework also increases the likelihood of college enrollment, especially for students from low-income backgrounds.[3] And as STEM careers grow at three times the rate of those in non-STEM fields, strong math skills are more essential than ever for unlocking future opportunities.[4]

Yet despite the clear and wide-ranging benefits of strong math foundations, student math achievement in the United States remains stagnant—or worse, it is in decline. While there have been modest gains in fourth grade math scores, overall performance on the National Assessment of Educational Progress (NAEP) still lags behind pre-pandemic levels.[5] International assessments paint a similarly troubling picture: More than a third of U.S. 15-year-olds are classified as "low performers" on the Programme for International Student Assessment (PISA), struggling with basic, real-world applications like comparing distances.[6] The Trends in International Mathematics and Science Study (TIMSS) further reveals widening post-pandemic achievement gaps between high- and low-performing students.[7]

This research summary explores the evidence behind effective math instruction. It examines the critical knowledge and skills teachers need to teach math well, the importance of high-quality teacher preparation, and the supports that both students and teachers need to achieve lasting success in math.

What do teachers need to know about math?

Content knowledge

In general, elementary students achieve more in math when taught by teachers with greater mathematics content knowledge.[8] Unfortunately, earning a bachelor's degree or completing a teacher preparation program does not guarantee that teachers will know the math they'll be expected to teach. A national survey found that few elementary teachers felt very well prepared to teach specific elementary mathematics topics, and the proportion who felt very well prepared declined between 2012 and 2018.[9] One study found that teacher candidates in one state performed poorly on the same topics that students in the state struggled with, although this study could not draw a causal link between teachers' knowledge and that of students.[10]

Another study found that many elementary teacher candidates had misconceptions about statistics and probability as they were about to enter student teaching (the culminating experience of most teacher preparation programs).[11] This sense of inadequate preparation has persisted for decades. In 2002 detailed surveys of elementary teachers in 60 school districts in Michigan and Ohio indicated that elementary teachers did not feel well prepared to teach the mathematics topics of the elementary level or slightly beyond:[12]

- In at least three-quarters of the 60 districts, less than half of first through third grade teachers considered themselves "very well prepared" to teach more than 60% of the topics. A little more than half of teachers in all districts in Michigan and Ohio felt "very well prepared" to teach just three topics (out of 28).
- In at least three-quarters of the districts, less than half of fourth and fifth grade teachers considered themselves "very well prepared" to teach more than 50% of the topics. At least 55% of teachers in all districts felt "very well prepared" to teach just three topics.

Most research suggests that teacher candidates' mathematics coursework—and content knowledge learned—yields benefits for their students. Several studies have demonstrated that teachers deliver stronger lessons on math topics that they learned in their teacher preparation programs.[13] A study of teacher preparation programs (both traditional and alternative) in New York City found that math courses in teacher preparation correlated with increased student achievement in math during the second year of teaching.[14] Another study found that not only the number of content courses, but also the types of courses (content, pedagogy, etc.) matter for building candidates' knowledge.[15] While most research points to a positive relationship between teacher candidates' math coursework and student outcomes, findings are not entirely uniform. For instance, one study did not find a significant link between the number of math education credits a teacher earned and their students' math achievement.[16]

Over the years, experts have coalesced around recommendations for the course time and content topics that aspiring elementary teachers need to prepare them to teach mathematics. In 2008 NCTQ convened a mathematics advisory group to inform the *No Common Denominator* report, which recommended that programs require the equivalent of about two-and-a-half courses (115 instructional hours) to address mathematics content and one course covering mathematics pedagogy (45 instructional hours).[17]

The Conference Board of the Mathematical Sciences's (CBMS) *The Mathematical Education of Teachers II* report recommends 12 semester hours, or four typical courses, in mathematics for elementary teacher candidates.[18] The recommendation includes both mathematics content and mathematics pedagogy. In dividing the 12 semester hours, the report assigns half the hours to numbers & operations and algebraic thinking, with the remaining half to additional algebra ideas, geometry & measurement, and data analysis & probability. Mathematics pedagogy is not separated from mathematics content when assigning instructional hours.

In 2021, NCTQ convened an expert advisory panel of 10 mathematics education experts to define key content and pedagogy areas for elementary mathematics teacher preparation and recommend instructional hour targets.

The panel recommended the following instructional hour targets:

- **Numbers and operations:** 39 hours
- **Algebraic thinking:** 24 hours
- **Geometry and measurement:** 27 hours
- **Data analysis and probability:** 17 hours
- **Mathematics pedagogy:** 49 hours

Pedagogy

A strong foundation in mathematical content knowledge generally leads to greater student achievement.[19] However, understanding content—numbers and operations, algebraic thinking, geometry and measurement, data analysis and statistics, and other areas— is only one component of effective math instruction. Teachers also need to learn how to apply mathematical content in their teaching through strong pedagogical knowledge—that is, knowing not just what to teach, but how to teach it.[20] In fact, improvements in teaching practice and student learning are associated with teachers understanding three concepts: mathematics content, how students gain mathematical understanding, and effective pedagogical practices.[21]

Historically the "rule method" of teaching, in which students memorized rules (e.g., $a^2+b^2=c^2$) and practiced using them, was the most common way to teach mathematics from colonial times until the early 19th century.[22] Since then, rote memorization has slowly given way to approaches that encourage students to develop both procedural fluency and conceptual understanding.

Conceptual understanding, procedural fluency, and application

Conceptual understanding can be defined as the "integrated and functional grasp of mathematical ideas," requiring students to understand the different contexts in which math concepts are useful and why they are important.[23] For example, conceptual understanding encompasses students being able to recognize that 1/2 and 2/4 are equivalent, representing the same percentage of a whole, despite having different numerators and denominators. Effective mathematics instruction depends on students developing a strong foundation of conceptual understanding, which in turn supports the development of procedural fluency.[24]

Procedural fluency is the "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently."[25] This includes skills like quickly calculating simple arithmetic, such as 2 + 7 = 9 or 4 x 4 = 16, in elementary years and simplifying expressions and graphing linear functions in later years. Procedural fluency serves as a foundation for executing mathematics mentally, on paper, or through the use of other technology.

Teachers can develop their students' procedural fluency by highlighting relationships between numbers and developing students' ability to reason. For example, activities that encourage students to break apart numbers, use visual models (e.g., number lines or arrays), or explain multiple ways to solve a problem help build a deep understanding of how and why math works. Put another way, building students' conceptual understanding supports their development of procedural fluency. Several studies highlight that students who learn these types of strategies perform better than students who learn other approaches, such as rote memorization. A 2015 analysis found that strategies such as subtraction as addition (e.g., when trying to solve 9 - 2, thinking "What plus 2 equals 9?") were more effective in promoting fluency in math than traditional drill-based methods.[26] A separate 2015 study examining multiplication fact fluency in third, fourth, and fifth grade classrooms also found that fluency was better achieved through strategy-based approaches (such as using multiplication by 2, 5, and 10 to solve unknown facts) than through drill-based memorization.[27]

Notably, too much emphasis on procedural fluency can lead to certain negative outcomes. A 2018 analysis found that students performed better in math when taught in classrooms that emphasized conceptual learning through strategies like inventing, analyzing, or proving rather than instruction focused on procedures alone.[28] And a 2019 analysis found that higher levels of procedural fluency with algebra were often correlated with greater misconceptions about certain concepts, providing further evidence that a balanced approach that considers both procedural fluency and conceptual understanding is critical to student proficiency in math.

The preceding research supports the National Council of Teachers of Mathematics's (NCTM) position that the way in which teachers teach fluency is critical. Teaching procedural fluency well requires emphasizing reasoning and decision-making as central to learning, helping students shift their mindset from "How did my teacher show me to solve this problem?" to "What are the strategies I've learned that can best solve this problem?"[29]

The final defining aspect of mathematical proficiency is application—the ability to solve a wide range of problems, across different contexts, by reasoning through and applying learned mathematical concepts.[30] This involves not just knowing the math, but being able to use it effectively and flexibly. Key elements of application include:

- **Strategic competence:** the ability to formulate, represent, and solve mathematical problems
- **Adaptive reasoning:** the capacity for logical thought, reflection, explanation, and justification
- **Productive disposition:** the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy[31]

Teaching strategies

Most states agree that conceptual understanding and procedural fluency are essential for effective math application. This consensus is reflected in the widespread adoption of the Common Core Standards for Mathematics, which emphasize all three aspects.[32] However, another debate has since emerged: whether students or teachers should take the lead in the learning process.

Student-led instruction

Student-led instruction involves actively engaging learners through strategies like connecting new information to prior knowledge and encouraging discussion among students.[33] In this environment, teachers act as facilitators, supporting students as they construct their own knowledge through exploration and reasoning.[34] This model emphasizes engagement, autonomy, and student-to-student interaction, aiming to deepen learning through active participation.[35] The role of the teacher is to be the facilitator of students leading instruction, akin to the "guide on the side."

Examples of student-led learning include:

- **Inquiry-based learning:** Students investigate questions, form hypotheses, and test them through observation or experimentation to discover new relationships and deepen understanding.[36]
- **Constructivism:** Students build knowledge through active engagement, reflection, and collaboration, connecting new ideas to prior experiences.[37]
- **Flipped classrooms:** Students receive most direct instruction outside of class, typically through videos or readings, while class time is used for hands-on activities, problem solving, and deeper discussion.[38]

- **Discovery-based learning:** Students learn through exploration, experimentation, and problem solving, which encourages critical thinking, creativity, and self-directed learning.[39]

Some researchers suggest that student-led instruction may positively impact student motivation, engagement, and learning outcomes.[40] For example, a review of 30 empirical studies on flipped classrooms found a positive effect on students' math learning. However, researchers noted that most studies on flipped classrooms have been conducted at the postsecondary level, with only about one-third taking place in traditional K–12 settings, and all of those at the middle and high school level.[41] A small study of two eighth grade pre-algebra classrooms found that students using inquiry-based learning, which encourages them to formulate and test hypotheses, outperformed peers in a traditional instructional setting. While the study found positive results, the researchers acknowledge its limited generalizability as it was conducted in a single school. They also note broader concerns raised in the field about inquiry-based learning, including its limited emphasis on procedural fluency,[42] the frequent need for teachers to supplement such curricula, and the significant scaffolding often required for it to be effective.[43] Notably, a 2004 review argues that decades of research have consistently shown unguided instructional approaches—like inquiry-based learning—to be less effective than teacher-directed instruction.[44]

Student-led instructional approaches may also support students in developing stronger conceptual understanding of mathematics by requiring students to connect what they are learning to prior knowledge. This might occur through open-ended tasks, collaborative problem solving, and guided discussions that prompt students to articulate their thinking. However, these connections do not occur automatically; teachers play a crucial role in designing learning experiences that help students surface and build upon their prior knowledge.[45]

Education scholars have noted that student-centered instruction tends to emphasize students' personal improvement and progress toward mastery, rather than outperforming their peers.[46] And research has shown that student-centered instructional approaches support students in developing a stronger conceptual understanding of mathematics by requiring them to connect what they are learning to prior knowledge.[47]

Teacher-led instruction

While recent research reveals positive findings associated with student-led learning, a long-standing body of research emphasizes the positive impact of direct, teacher-led instruction. In fact, explicit or direct instruction—where teachers guide learning and serve as the primary source of information—has been shown to benefit students across a range of learning needs.[48]

Examples of teacher-led instruction include:

- **Direct instruction:** Explicit, systematic instruction of specific skills or concepts. Teachers model procedures, guide students through practice, and provide immediate feedback to ensure understanding before moving on to more complex material.[49]
- **Explicit instruction:** Teachers clearly communicate learning goals, directly teach skills or concepts, and frequently check for student understanding to guide instruction.[50]

In math, teacher-led instruction typically involves direct teaching and demonstration of mathematical procedures, followed by students practicing similar problems independently.[51] A substantial body of evidence supports the positive impact of teacher-led instruction. A 2018 meta-analysis of 328 studies over 50 years found a strong positive relationship between student achievement across all subjects and direct instruction. The positive impacts of direct instruction were consistent across a wide range of student populations, including students from high-poverty backgrounds and those receiving special education services. These gains were also sustained over time, with minimal decline in follow-up studies. Notably, students who received more intensive exposure to direct instruction—whether through longer participation, extended daily instructional time, or an earlier start—experienced even greater academic gains.[52]

Teacher-led techniques have shown particularly significant gains for at-risk students. For example, a 2007 study on middle school math achievement revealed that students who had previously failed state assessments significantly improved their fraction skills and demonstrated increased on-task behaviors when taught via direct instruction.[53] A 2014 analysis examining the effectiveness of different instructional practices on first grade math achievement found that teacher-directed instruction was more beneficial than other teaching techniques for students with math difficulties.[54] Notably, researchers in the 2014 study found no difference between student- and teacher-centered instruction for students without math difficulties.

What is the role of teacher prep?

Most new teachers receive training through a traditional teacher preparation program affiliated with an institution of higher education (though in some states, there has been a rise in alternative certification programs or alternative pathways to the classroom that do not require pre-service preparation). Here we examine research on the effect of teacher preparation programs as well as policy levers that can help improve their effectiveness.

State standards for teacher preparation programs

Teacher preparation standards describe the knowledge and skills state leaders have determined aspiring educators need to be effective. These standards serve as a foundation for educator preparation programs (EPPs) to design curricula, instruction, and assessments that ensure future teachers are well equipped for the classroom.

For prospective elementary teachers, mathematics coursework must go beyond general math classes that apply to all college students, focusing instead on deep, specialized understanding of the mathematics concepts they will teach.[55]

Leading mathematics and mathematics education organizations have identified key content areas that EPPs should teach. CBMS recommends that aspiring elementary teachers complete 12 semester-credit hours in "elementary mathematics content" covering numbers and operations, algebra, measurement and data, and geometry.[56] Similarly, NCTM recommends aspiring teachers take at least three college-level mathematics courses in the content essential to elementary grades in addition to instruction on pedagogy.[57] The Mathematical Education of Teachers II (MET II) study, aligning with the Common Core State Standards for students, further emphasizes the need for elementary teachers to be well versed in counting and cardinality, operations and algebraic thinking, numbers and operations, measurement and data, and geometry, with additional connections to middle-grade mathematics.[58]

NCTQ was not able to identify any studies that examined the relationship between states' math standards for preparation programs and prep programs' adherence to those standards or their impact on teachers' knowledge or their students' outcomes. Clearly this is an area for more research.

Prep program approval

Teacher preparation programs must receive state approval to operate, making state evaluations a critical tool for assessing whether these programs effectively implement standards set by the state and equip aspiring teachers with the knowledge, skills, and experiences needed for classroom success. These evaluations serve multiple purposes for various stakeholders. They support continuous program improvement by identifying strengths and areas for growth; hold programs accountable for meeting established standards or else face closure; and have the potential to provide valuable consumer information about program quality to prospective teacher candidates, school districts hiring new educators, and policymakers shaping education policy.[59]

In most states, the responsibility for reviewing and approving educator preparation programs falls to one of three entities: the state department of education (25 states), the state board of education (12 states), or the board of regents (5 states). In eight states, this role is assigned to a distinct state-specific entity or governance body.[60] Many states contract with external accrediting bodies to conduct or inform their program review.

There is limited research examining the long-term impact that strong program approval processes have on outcomes like student achievement or educator effectiveness, which highlights the benefit of strong evaluation processes. A 2021 study in Massachusetts found a direct link between the performance rating of preparation providers (which could be institutions that house multiple programs) and the future effectiveness of their graduates in classrooms—both in evaluation ratings in their new schools and their contributions to student learning.[61]

A 2024 report from the National Academy of Education identifies six key evidence-based features of teacher preparation programs associated with ensuring that candidates have the “knowledge, skills, and dispositions” of high-quality teaching.[62] Several of these features are relevant to math instruction, including program coherence and alignment, curriculum content, and instructional methods.

Coursework

There is broad expert consensus that elementary teacher candidates need substantial mathematics content coursework to be well prepared, as discussed in the content knowledge section above. Extensive research indicates that effective mathematics coursework for elementary teachers blends both content and methods. This approach not only provides a solid foundation in elementary mathematics, but also serves as a crucial link to classroom instruction.[63] Teachers who possess specialized content knowledge are better equipped to design lessons that integrate mathematics and science, use manipulatives effectively, and employ student-centered teaching approaches in mathematics.[64]

A wide range of experts recommend that educator preparation programs be structured to encompass both subject matter knowledge—including common and specialized content knowledge—and pedagogical content knowledge, which covers understanding of content, students, and teaching methods.[65]

Specialized content knowledge

Elementary teachers need to grasp more than the mathematical knowledge and skills required in student curricula. They need to master what is unique to teaching, the “unpacked mathematical knowledge” needed to make “content visible to and learnable by students."[66] This includes being able to create and adapt representations of math problems to suit specific instructional purposes, to both carry out and explain algorithms for solving problems, and to conduct error analysis to uncover and address student misconceptions.[67] One study, which examined whether and how elementary teachers improved their mathematical content knowledge through participation in a state-funded professional development institute, suggests that teachers may learn more effectively when they "engage with mathematics in ways that afford them more opportunities to explore and link alternative representations, to provide and interpret explanations, and to delve into meanings and connections among ideas."[68]

Licensure tests

Most states require prospective teachers to pass a licensure exam before they can earn a certification and lead a classroom. While it can be difficult to isolate the specific impact of licensure exams on long-term teacher and student outcomes, especially as different states require different tests, most research generally demonstrates that passing such assessments has a moderate but statistically significant relationship with effectiveness in the classroom.[69]

Studies consistently find that teacher licensure exam scores are positively associated with student achievement. One study found that, while years of teacher experience have a positive relationship with middle school student test scores, teacher licensure exam scores had an even stronger association.[70] Results from a longitudinal study demonstrate teacher licensure test scores have positive impacts on students' test scores in grades 3–5 in North Carolina.[71] In Florida, researchers found a positive relationship between certification exam performance and teachers' classroom effectiveness, noting that failing the state certification exam one or more times was associated with lower student test scores in 4th–8th grade math and 6th–8th grade reading.[72]

In Massachusetts, higher scores on the Massachusetts Tests for Educator Licensure (MTEL) were also tied to stronger student outcomes.[73] Similarly, research on the Praxis exams, which are used in 46 states,[74] shows that teachers' curriculum-related knowledge—more so than their general content knowledge—was a strong predictor of student achievement.[75] Research from Arkansas also confirms a positive relationship between teachers' Praxis II scores and student achievement.[76]

What supports do in-service teachers need?

Professional learning

Research has shown that most professional development is quite expensive for districts: TNTP's 2015 report on professional learning found that large districts may spend as much as $18,000 per teacher per year on teacher development. The same report also found that most teachers do not improve from year to year, and when they do, no amount or combination of professional development opportunities is linked to the growth.[77]

This failure to see a return on investment from professional learning highlights a lack of understanding around what aspects of professional learning are associated with improved outcomes for teachers and students, because well-designed and implemented professional learning can help teachers improve their practice.

A meta-analysis synthesizing 46 math and science studies found that professional learning can positively impact student achievement: Teachers who participated in professional learning focused on content and pedagogical knowledge or classroom instruction (student engagement, class time management, etc.) scored higher on measures of knowledge and classroom instruction than control group teachers.[78] The same study found that classroom instruction practices were positively associated with greater student achievement: Each standard deviation (SD) improvement in classroom instruction was associated with a 0.24 SD improvement in student achievement scores, the equivalent of a student improving from the 50th to the 59th percentile. Notably, content and pedagogical knowledge was not linked to improved student outcomes, yet researchers noted that interventions that emphasized both types of professional learning simultaneously had the largest positive effect on student math and science achievement.[79]

Another study examined a two-year fellowship program that included opportunities for leadership development, classroom instructional coaching, and a specific focus on English language arts (ELA) and math content and pedagogical knowledge. It found that participating scholars experienced greater student achievement gains than nonparticipating schools. Based on those findings, researchers identified four factors they believe are linked to improved teacher practice and student learning, including the proportion of participating school staff (saturation), whether participants joined individually or as a team (enrollment type), the length of the professional development program, and the involvement of district-level staff.[80]

Coaching

Instructional coaches

Research has established that high-quality instructional coaching for teachers can make a substantial difference in both the quality of a teacher's practice and student outcomes. A 2018 meta-analysis of 60 causal studies demonstrated that instructional coaching improves teacher effectiveness to a degree comparable to the difference between a novice teacher and one with 5 to 10 years of experience.[81]

The Association of Mathematics Teacher Educators (AMTE) outlines three essential areas of expertise for mathematics coaches (often called specialists): content knowledge for teaching mathematics, pedagogical content knowledge, and leadership skills, which in this context refers to coaches taking on collegial, non-evaluative roles within their communities.[82] Additionally, numerous studies highlight the competencies that math instructional coaches should possess, including a deep understanding of instructional practice and theory, the ability to offer differentiated experiences for teachers, and the capability to create long-term goals, among others.[83]

Research examining the impact of instructional coaching on mathematics achievement is limited but generally positive. A three-year randomized control trial found that elementary mathematics coaches improved student achievement in grades 3–5 but only after the first year of implementation. Coaches in this study participated in extensive coursework on math content, pedagogy, and coaching before and during their first year, underscoring the necessity of providing strong support to ensure coaches benefit students and teachers.[84]

A 2020 analysis of an instructional coaching program in Tennessee found that coaches who received targeted training developed stronger coaching practices, such as deep, specific pre-lesson planning conversations with teachers, which predicted improvements in teaching, including increased opportunities for students to engage in conceptual mathematical thinking.[85] Additionally, a 2010 study on the impact of a mathematics coaching program for teachers of fourth grade students in low-performing schools found that the number of students performing below average on the state's standardized assessment significantly decreased across all five content areas: measurement, numbers, algebra, data, and geometry (although this study could not prove a causal relationship).[86]

Well-designed coaching programs provide numerous benefits to teachers and students beyond improving teacher instructional practices and student achievement. A 2018 analysis of a nationally representative sample of K–12 teachers found that the presence of a curriculum coach was associated with reduced early-career turnover, particularly in urban areas.[87] Furthermore, a randomized control trial of a two-year teacher coaching program at the secondary level, which focused on fostering relationships between students and teachers, successfully eliminated racial disparities in student discipline referrals.[88]

Notably, researchers have found that the effectiveness of coaching programs declines as programs grow larger, an indication that such programs face scaling challenges. The previously cited meta-analysis found that when coaches are assigned more than 100 teachers, coaching effectiveness is significantly reduced, highlighting that a high teacher-to-coach ratio could threaten the fidelity of implementation and negatively impact a coach's overall effectiveness.[89] Another study found significant variation in the amount and type of coaching that elementary mathematics coaches deliver and that the impact of coaching can be significantly reduced when coaches are relegated to more administrative tasks, rather than working directly with teachers.[90]

What supports do students need to succeed?

High-quality instructional materials

Curricula—often called instructional materials—are the core materials that teachers use to deliver instruction. High-quality curricula are core materials that have been vetted by the state or a designated partner to ensure they align to state standards, support building content knowledge, promote rigorous, grade-level learning, and are grounded in up-to-date research.

Several studies affirm that high-quality instructional materials (HQIM) have a significant, positive impact on student achievement.[91] In fact, the difference in student learning between high- and low-quality curricula can be greater than the difference between a novice teacher and one with three years of experience.[92]

In a randomized trial, middle school math teachers who received access to HQIM saw statistically significant improvements in student performance compared to teachers who did not receive such materials, with even greater gains for newer teachers.[93] Similarly, a large-scale comparative study of four elementary math curricula found that the specific curriculum used made a significant difference in student achievement.[94]

Yet despite this strong evidence, many students—particularly those in historically underserved communities—still do not have guaranteed access to high-quality math curricula. Schools in these communities are more likely to teach students with mediocre or low-quality materials, deepening existing inequities.[95]

Access to high-quality instructional materials alone, however, is not enough. In fact, one study saw no differences in the average fourth and fifth grade math achievement between schools that used high- and low- quality math textbooks. The authors concluded that to see instruction improve—and student outcomes along with it—curriculum alone is not enough.[96]

Teachers need ongoing, curriculum-aligned professional development on how to use HQIM effectively. One study found that nearly 60% of the potential impact of adopting HQIM depends on teachers effectively adapting their instructional practices to align with the materials.[97] And in the previously cited study of middle school math teachers, those who received support to implement new curricula saw greater student gains than those who only received the materials.[98] Teachers and school leaders back up the research. In a nationally representative survey of thousands of teachers and leaders, the alignment of professional learning with math curricula was a top priority for principals and district leaders. Teachers found curriculum-based professional learning most helpful when it was focused on the specific curriculum they use in their classrooms.[99]

Student interventions

Math interventionists and similar supports

While this research summary primarily focuses on math coaches and specialists for teachers, schools may also deploy similar roles and use similar titles for people who work directly with students. Adding to the potential confusion, some coaches work with both teachers and students. For clarity we will use the term *math interventionist* to refer to educators who work primarily with students, though terms like *coach* or *specialist* may be used interchangeably for teachers.

An analysis aimed at evaluating the impact of two factors (curriculum implementation fidelity and interventionist content knowledge) on student outcomes for 4th–6th graders in a specific intervention program revealed that students achieved better test scores when interventionists adhered closely to the program's scripted routines, such as lesson pacing, modeling, and following script instructions. Interestingly, higher levels of math content knowledge in interventionists did not significantly influence students' math achievement scores.[100]

A separate study compared the effects of two distinct math intervention programs: one focused on improving students' conceptual understanding and the other providing extended time for students to work on core curriculum materials. Both programs trained existing classroom teachers to deliver small-group math interventions to students.

Students who participated in either intervention demonstrated stronger math outcomes than those who did not receive any additional support.[101]

English learners

Given the significant rise in students identified as English or multilingual learners over the past decade, it is critical to support their mathematics learning and address any challenges they face.[102] Research suggests that instruction for English learners should have two key characteristics: It should view language knowledge as a resource and a strength, not a deficit, and it should emphasize high-quality, rigorous content and instruction across all academic areas, not just English language acquisition.[103] A 2018 National Academies of Science consensus report echoes these two characteristics of effective math instruction for ELs: an emphasis on overall academic achievement (not only on learning English) and recognition of the meaning-making resources students bring to the classroom.[104]

A growing body of research explores the connection between English proficiency and math. A 2018 literature review of 75 empirical studies provides substantial evidence of the interconnectedness between English proficiency and math achievement.[105] Several studies in this meta-analysis indicate that English learners with higher language proficiency perform better in mathematics.[106] In a 2010 study, researchers explored differences among students in achievement on a literacy-based mathematics performance assessment that required students to demonstrate their understanding through writing, rather than through multiple-choice questions or other formats. The study found that fully English proficient students who came from higher socioeconomic backgrounds performed better on the assessment than learners who had not yet mastered English and were lower income. However, it found no statistically significant difference among lower-socioeconomic students based on their English proficiency.[107] The study ultimately concludes that literacy-based math assessments do not provide equal treatment for English learners and place them at an academic disadvantage.

A meta-analysis of two types of language-focused interventions targeting math vocabulary for English learners (single-student and group) found that both interventions improve student performance. However, single-student interventions prove more effective due to the emphasis on one-on-one instruction.[108] A study examining the impact of native language intervention, where students receive math support in their native language based on their math comprehension level, found that it increased and sustained students' word problem solving levels.[109]

Special education

Students with disabilities need consistent access to evidence-based instruction that supports both conceptual understanding and procedural fluency aligned with grade-level content standards.[110] Both the Council for Exceptional Children (CEC) and AMTE emphasize that students with disabilities should be taught by educators who possess strong content knowledge and pedagogical expertise—just as expected for all students.[111]

Research on effective math instruction for students with learning disabilities is limited. However, a 2009 meta-analysis of 41 studies conducted between 1971 and 2006 identified several instructional and curriculum design strategies that significantly improved math proficiency for these students. Among the most effective were explicit, teacher-directed instruction and the use of heuristics (general problem-solving strategies).[112]

In this study, *explicit instruction* referred to a structured teaching approach that included three key components: the teacher modeled a step-by-step strategy for solving a specific type of problem, the strategy is problem specific rather than being a general problem-solving approach, and students were expected to reproduce the demonstrated steps to solve similar problems. When faced with a word problem involving fractions, a problem-specific approach may involve the teacher providing explicit, step-by-step guidance tailored to the structure of that particular problem. For example, if the problem requires finding a fraction of a mixed number, the teacher might walk students through converting the mixed number to an improper fraction, identifying the correct operation (e.g., multiplication), executing the calculation, and then interpreting the result within the context of the problem.

Heuristics, by contrast, were more general. An example heuristic might be, "Read the problem. Highlight the key words. Solve the problem. Check your work." While less specific than explicit instruction, heuristics still provided a structured framework to guide students through problem solving. Other strategies, such as encouraging students to think aloud and employing visual representations, also saw statistically significant impacts on student proficiency.[113]

Access to advanced coursework

Access to advanced coursework serves as a critical stepping stone on the path to success in math- and science-based careers. As a result, there is growing interest in policies that expand access to these courses, including offering Algebra I in eighth grade or using indicators like assessment scores to automatically enroll students in advanced coursework like Advanced Placement courses. This can be particularly beneficial for students of color, as research demonstrates that they enroll in advanced coursework at significantly lower rates than their white peers, despite being equally likely to succeed in such courses.[114]

Limited research has shown that students who gain access to advanced coursework are more likely to continue excelling in advanced mathematics, opening doors to future academic and career opportunities.[115] Using automatic enrollment policies, which automatically assign students to advanced math courses if they perform well on objective measures like test scores, rather than relying on teacher recommendations or student decisions to opt into courses, may help expand access. But given that automatic enrollment policies are a relatively new policy—currently implemented in only six states—research on their long-term impact remains limited. However, early results from North Carolina, which automatically enrolls students in grades 3–12 into advanced learning opportunities like gifted pathways (grades 3–5) or advanced coursework (grades 6 and up) when they perform at a level 5 on the state assessment, are promising: Since the policy's implementation in 2018, enrollment in advanced math courses has risen significantly across all student subgroups, although there is no research yet about student outcomes.[116]

Increasing students' opportunities to take algebra during eighth grade (e.g., by requiring every middle school to offer an eighth grade algebra course), rather than requiring students to wait until high school, is another strategy that has seen increased interest. Although research on the long-term impacts of early algebra access remains limited, emerging analyses and state-level examples offer some insight. Taking algebra before high school has been linked to stronger high school math achievement and completion of advanced coursework.[117]

A study of North Carolina's 10 largest districts found that efforts to increase eighth grade Algebra I enrollment (primarily through district-level policy changes encouraging earlier placement into advanced math) had mixed results: High-performing students experienced moderate benefits, while students in the bottom 60% of the achievement distribution saw negative outcomes. These lower-performing students scored worse on the Algebra I end-of-course exam, were more likely to repeat the course, and were less likely to pass geometry by 11th grade.[118] In contrast, a smaller study of a single district compared students who took Algebra I in eighth grade to their peers (with similar academic proficiency) who took algebra in high school, finding that both groups of students performed similarly on later assessments—including high school exams and the SAT math section—but those who took Algebra I in eighth grade were more likely to stay on the advanced math track and attend college than their peers who took algebra later.[119]

Several states have implemented policies to increase access to eighth grade algebra with uneven results. Take for example Minnesota's long-standing policy requiring all students to take Algebra I by eighth grade. Although there has been a modest uptick in the number of students reaching calculus (rising from 1.25% to 1.76%), overall participation in advanced math courses still trails that of states without such universal mandates.[120]

A similar pattern emerged in California, where a 2015 study examined the effects of making Algebra I the standard accountability test for eighth grade math. Though the goal was to broaden access to advanced coursework, the policy backfired in some cases—particularly in larger districts—where students were placed into classes they weren't ready for, ultimately resulting in lower achievement.[121] As research and state practices continue to evolve, ensuring access to and success in Algebra I during middle school will likely hinge on clearly defining when students should be enrolled, how readiness is determined, how students are grouped within Algebra I classrooms, and what instructional supports are needed to promote their success.[122]

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