What to bring to class:
Ask students to bring PM 4A and 5A.

8.3 Integers as numbers

Integers are numbers because they behave like them:

(1) Commutative property $-3 + 5 = 5 + -3$, $(-3)\,5 = 5\,(-3)$

(2) Assoc. Prop $(-3 + -2) + 7 = -3 + (-2 + 7)$, $-3 \times (-2 \times 7) = (-3 \times -2) \times 7$ } check using Rules!

(3) Distributive Prop. $3\,(-4 + -2) = 3 \times -4 + 3 \times -2$

(4) Identities $-3 + 0 = -3$, $-5 \quad 1 = -5$.

And a new property which is important to integers:

(5) Additive inverse property: For each integer a there exists an integer called the opposite of a, denoted by $-a$, which satisfies

$$a + -a = 0.$$

[SAY: Additive Identity $\longleftrightarrow$ Describes 0
Mult. Identity $\longleftrightarrow$ Describes 1 } we already have prop. for 0, 1. This is why we exclude from def of prime numbers.
Mult. Inverse $\longleftrightarrow$ Describes fractions
Additive Inverse $\longleftrightarrow$ Describes integers!]

Summary

(1) Created "opposite numbers"

(2) Made interpretations so we could +, −, ×, ÷ them. Interpretation summarized by Rules 1–6.

(3) Integers satisfy properties ($\Rightarrow$ integers are numbers)

(4) Special property associated to integers.

We need to show:

(5) Properties ⇒ Rules 1-6
"Interpretations"

(5) means that we cannot develop a different set of interpretations, these rules are <u>forced</u> upon us by the properties!

Theorem: Rules 1-6 follow from the arithmetic properties.

<u>Rule 1</u>: $-(-a) = a$ [uses Decompress/ Analyze/ Compress proof w/ Add an appropriate 0.]

$-(-a) = 0 + -(-a)$ Additive Identity

$= (a + -a) + -(-a)$ Additive Inverse

$= a + [-a + -(-a)]$ Associative Property

$= a + 0$ Additive Inverse

$= a$ Additive Identity

<u>Def 8.1.3</u> $a + -b = a-b$

$a - b = \_\_$ means $a = b + \_\_$

Add $-b$ to both sides:

$a + -b = -b + (b + \_\_)$

$= (-b + b) + \_\_$ Associative Property

$= 0 + \_\_$ Additive Inverse

$= \_\_$ Additive Identity

$= a - b$ Substitution

[so def is really a rule]

<u>Rule 2</u>: $a - -b = a + b$ Done in last HW set
[Follows from R1 and Def 8. 1. 3]

Rule 3: $-a \cdot b = -(a \cdot b)$

1st

Ask

$-3 \cdot 4 = -3 \cdot 4 + 0$ __

$= -3 \cdot 4 + [3 \cdot 4 + -(3 \cdot 4)]$ __

$= [-3 \cdot 4 + 3 \cdot 4] + -(3 \cdot 4)$ __

$= (-3 + 3) \cdot 4 + -(3 \cdot 4)$ __

$= 0 \cdot 4 + -(3 \cdot 4)$ __

$= 0 + -(3 \cdot 4)$ __

$= -(3 \cdot 4)$ __

$= -(12) = -12$

2nd

$-a \cdot b = -a \cdot b + 0$

$= -ab + [ab + -(ab)]$

$= [-a \cdot b + ab] + -(ab)$

$= (-a + a) \cdot b + -(ab)$

$= 0 \cdot b + -(ab)$

$= 0 + -(ab)$

$= -(ab)$ ✓

Rule 4: (using Already verified Rules)

$-a \cdot -b = -(a \cdot -b)$ Rule 3

$= -(-b \cdot a)$ Commutative Property

$= -(-(ba))$ Rule 3

$= ba$ Rule 1

$= ab$ Commutative Property

Rule 5: $-a \div -b = a \div b$

what?

$-a \div -b =$ __ means $-a = -b \cdot$ __

Take the opposite of both sides:

$-(-a) = -(-b \cdot$ __ $)$

$\|$

$a = b \cdot$ __

$\updownarrow$

$a \div b =$ __ $= -a \div -b$!

Def. (Ordering) $a \leq b \Leftrightarrow b - a$ is positive or zero.

Order Rules

1. $a \leq b \iff a + c \leq b + c$

2. $a \leq b \iff \begin{cases} ac \leq bc & c > 0 \\ ac \geq bc & c < 0 \end{cases}$

Proof of Order Rule 1

$a + c \leq b + c \iff (b + c) - (a + c)$ is positive or zero.

$\iff b + c + -a + -c$ is pos. or 0 — HW Prob

$\iff b + -a + (c + -c)$ is pos. or 0 — Any order

$\iff b - a + 0$ is pos. or 0 — Add inv, R2

$\iff b - a$ is pos. or 0 — Add id.

$\iff a \leq b.$ — ■

HW Read 8.3 Do HW set 36