What to bring to class:
Ask students to bring PM 4A and 5A.

6.6 Fractions as Numbers

[SAY: Understanding fraction arithmetic becomes increasingly important as students prepare for algebra.]

Fraction aritmetic is developed using many artihmetic problems like the following:

Ex HW set 29. Prob 2d

$$\left[\left(\tfrac{1}{4}\cdot\tfrac{3}{4}\right)+\left(\tfrac{2}{3}\div\tfrac{4}{3}\right)\right]\div\tfrac{11}{12}=\left(\tfrac{1}{4}\cdot\tfrac{3}{4}+\tfrac{2}{3}\cdot\tfrac{3}{4}\right)\cdot\tfrac{12}{11} \quad \text{R5}$$

$$=\left(\tfrac{1}{4}+\tfrac{2}{3}\right)\cdot\tfrac{3}{\not{4}}\cdot\tfrac{\not{12}^{3}}{11} \quad \text{distributive Prop.}$$

$$=\left(\tfrac{3}{12}+\tfrac{8}{12}\right)\cdot\tfrac{9}{11} \quad \text{R1, R4}$$

$$=\tfrac{\not{11}}{{}_4\not{12}}\cdot\tfrac{\not{9}^{3}}{\not{11}} \quad \text{R2}$$

$$=\tfrac{3}{4} \quad \text{R4, R1}$$

Students do rest of the problems in HW set 29, Prob 2. (in groups of 2, 5 min.)

Fraction Arithmetic – mathematics

[SAY: We used models and interpretations to generate the 5 rules of fractions. Is it possible to use different models and interpretations to generate different (but valid) fraction rules? Then, for instance, each country or classroom could have a different way to add fractions!]

Recall from § 4.2

"All indentities (in particular, the arith. rules) can be derived from the arith. properites."

Thus

Arith. properties $\Rightarrow$ Rules 1–5 $\Rightarrow$ fraction arithmetic.

We are <u>forced</u> to make these rules –

they <u>do not</u> depend upon the models or interpretations.

To show this, we assume there is a set called "fractions" and

- There is some way to + and X them
- They satisfy the arithmetic properties (Any order, dist, identity)

<u>The Multiplicative inverse property</u>

For each nonzero fraction x there is a unique fraction called the inverse, $\frac{1}{x}$ such that

$$x \cdot \frac{1}{x} = 1.$$

Fraction: $1, 2, 3, 4, \ldots\ldots \frac{3}{4}$

$\updownarrow \updownarrow$

Inverse: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\ldots \frac{1}{\frac{3}{4}}$

($\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$: fractional unit)

<u>Def</u> Each fraction can be represented by a multiple of a fractional unit:

$$\frac{a}{b} = a \cdot \frac{1}{b}.$$

To prove "properties ⇒ rules" we need the following lemma:

<u>Lemma</u> Assuming only arithmetic properties,

$$\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab} \qquad \text{for } a, b \neq 0.$$

[SAY: The multiplication of 2 fractional units is again a fractional unit.]

<u>Proof</u>

$$\begin{aligned}
\tfrac{1}{a} \cdot \tfrac{1}{b} &= \tfrac{1}{a} \cdot \tfrac{1}{b} \cdot 1 && \text{mult. identity} \\
&= \tfrac{1}{a} \cdot \tfrac{1}{b} \cdot (ab)\tfrac{1}{ab} && \text{mult. inverse} \\
&= (a \cdot \tfrac{1}{a})(b \cdot \tfrac{1}{b}) \cdot \tfrac{1}{ab} && \text{Any order} \\
&= 1 \cdot 1 \cdot \tfrac{1}{ab} && \text{mult. inv.} \\
&= \tfrac{1}{ab} && \text{mult. identity}
\end{aligned}$$

<u>Thm</u> Rules 1–5 follow from the def. of fractions and the arithmetic property.

<u>Proof</u>

<u>Rule 1</u> $\frac{an}{bn} = \frac{a}{b}$

$$\frac{an}{bn} \underset{\text{def.}}{=} (an)\frac{1}{bn} \underset{\text{Lemma}}{=} an \cdot \frac{1}{b} \cdot \frac{1}{n} \underset{\text{any order}}{=} (a \cdot \tfrac{1}{b})(n \cdot \tfrac{1}{n})$$

$= \frac{a}{b} \cdot 1$ def of fractions, mult. inverse

$= \frac{a}{b}$ mult. id.

<u>Rule 2</u>

$$\frac{a}{b} + \frac{c}{b} \underset{\text{def}}{=} a \cdot \frac{1}{b} + c \cdot \frac{1}{b} \underset{\text{dist. prop.}}{=} (a + c)\frac{1}{b} \underset{\text{def.}}{=} \frac{a+c}{b}$$

<u>Rule 3</u> $a \div b = \frac{a}{b}$

def. of div.

$a \div b = x \Leftrightarrow a = bx$. Multiply by $\frac{1}{b}$

$$a \cdot \frac{1}{b} = \frac{1}{b}(bx) = (\frac{1}{b} \cdot b)x = 1 \cdot x = x$$

any order mult. inv. mult. id.

so $a \div b = x = a \cdot \frac{1}{b} = \frac{a}{b}$

def. of frac.

<u>Rule 4</u> HW

<u>Rule 5</u> By def, $\frac{a}{b} \div \frac{c}{d} = x \Leftrightarrow \frac{a}{b} = \frac{c}{d}x$.

Then

$$\frac{a}{b} \cdot \frac{d}{c} = \frac{d}{c}(\frac{c}{d}x) = \frac{dc}{dc} \cdot x = (dc \cdot \frac{1}{dc})x$$

any order R4 def of fractions

$= 1 \cdot x$ mult. inverse

$= x$ mult. id.

Thus $\frac{a}{b} \div \frac{c}{d} = x = \frac{a}{b} \cdot \frac{d}{c}$

Note: $\frac{1}{\frac{a}{b}} = 1 \div \frac{a}{b} = 1 \cdot \frac{b}{a} = \frac{b}{a}$

Rule 3 R5 Mult. id.

"Inverses are the same as reciprocals!"

HW Read § 6.6. Do HW set 29. Bring Primary math 5A & 6A to next 3 lectures.