What to bring to class:
Ask students to bring PM 4A and 5A.

6.4 Dividing fractions

Ultimate goal: "invert and multiply" rule

But must explain concepts

- What division of fractions means
- How to divide

} SAY: long time on this!

using models, interpretations, and word problems.

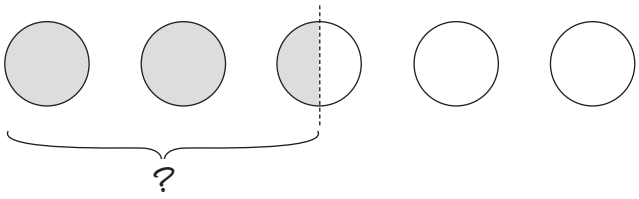
Teaching Sequence

Case 1: Whole ÷ Whole Review

Ex 1 2 girls share 5 cookies equally. How much did each get?

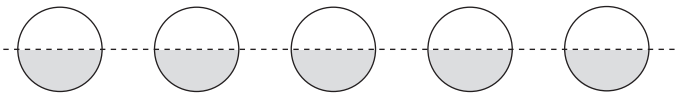
PD or MD? "5 is 2 groups of what?"

Either:



$5 \div 2 = 2\frac{1}{2}$

or



$5 \div 2$ = shaded portion = 5 half cookies = $5 \times \frac{1}{2} = \frac{5}{2}$

Answers are equivalent: $2\frac{1}{2} = \frac{5}{2}$. But 2nd viewpoint is starting point of fraction division!

$5 \div 2 = 5 \times \frac{1}{2}$

Case 2: Fraction ÷ Whole

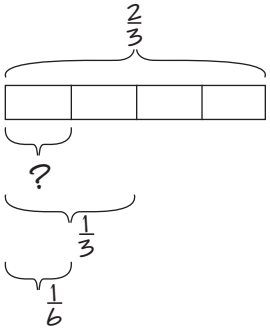
Partition fraction into groups.

Ex 2 4 boys shared $\frac{2}{3}$ quart of juice equally.

How much did each get?

PD or MD? "$\frac{2}{3}$ is 4 groups of what?"

T.S.



4 units =

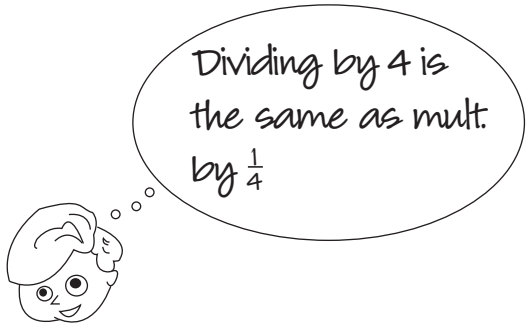
1 unit = $\frac{2}{3} \div 4 = \frac{1}{6}$.

Each got $\frac{1}{6}$ quart.

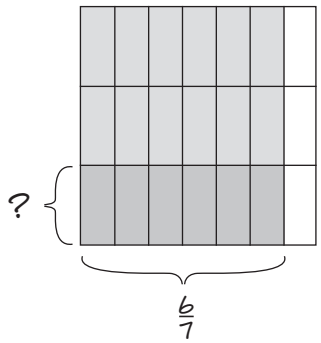
Note: $\frac{2}{3} \div 4 = \frac{1}{4}$ of $\frac{2}{3}$ (by diagram)

$= \frac{1}{4} \times \frac{2}{3}$

$= \frac{2}{12} = \frac{1}{6}$



Ex 3 $\frac{6}{7} \div 3$ using Area model.



Model shows:

$\frac{6}{7} \div 3 = \frac{6}{21} = \frac{2}{7}$

Abstractly:

$\frac{6}{7} \div 3 = \frac{1}{3}$ of $\frac{6}{7} = \frac{1}{3} \times \frac{6}{7}^{2} = \frac{2}{7}$

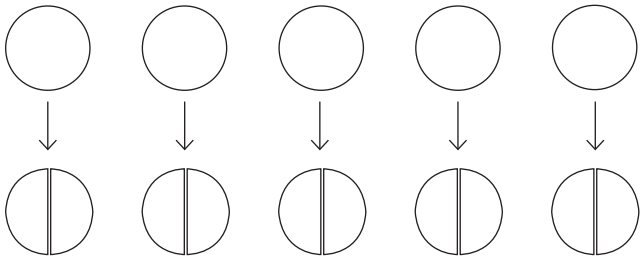
Note: $\frac{6}{7} \div 3 = \frac{6}{7} \times \frac{1}{3}$

Case 3 Whole ÷ fraction

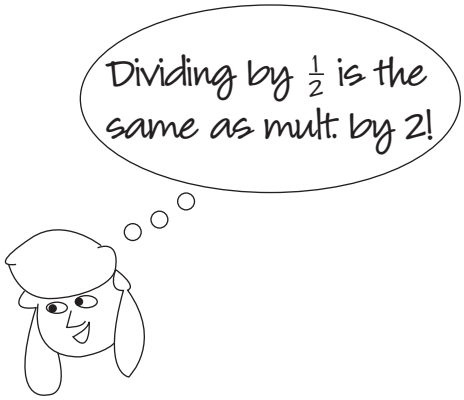
* Conceptually hardest case. Use models and word prob.

Ex 4 Jill bought 5 oranges. She cut each into $\frac{1}{2}$ pieces. How many halves did she have?

PD or (MD)? 5 is how many $\frac{1}{2}$'s?



10 pieces

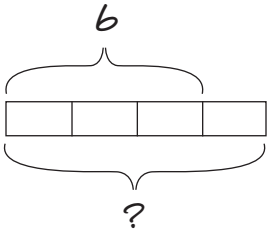


Hence $5 \div \frac{1}{2} = 10$.

Ex 5 Jim decided to walk to Jill's house from his. After 6 blocks he was $\frac{3}{4}$'s of the way. How far apart are their houses?

(PD) or MD? "6 is $\frac{3}{4}$ of what?"

T.S.



3 units = 6

1 unit = 2

4 units = 8

She lives 8 blocks away.

Abstractly: $6 \div \frac{3}{4} = (6 \div 3) \times 4 =$

$= (\frac{1}{3} \text{ of } 6) \times 4$

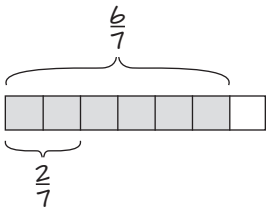
$= 6 \times \frac{1}{3} \times 4$

$= 6 \times \frac{4}{3} = \frac{24}{3} = 8.$

Case 4: fraction ÷ fraction

Ex 6 $\frac{6}{7} \div \frac{2}{7}$ using M.D.

$\frac{6}{7}$ is how many $\frac{2}{7}$'s?



Shows $\frac{6}{7} \div \frac{2}{7} = 3$

total per group # of groups

In general

Fracion Rule 5: $\frac{a}{b} \div \frac{c}{b} = a \div c$ (or $\frac{a}{c}$ using FR.3)

Note that this leads to the common " invert and multiply" rule.

Ex $\frac{1}{4} \div \frac{2}{3}$

Abstractly: $\frac{1}{4} \div \frac{2}{3} = \frac{3}{12} \div \frac{8}{12} = 3 \div 8 = \frac{3}{8} = \frac{1}{4} \times \frac{3}{2}$

Rule 1 Rule 5 Rule 3 Rule 4

More abstractly:

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bd} \div \frac{bc}{bd} = ad \div bc = \frac{ad}{bc} = \frac{a}{b} \times \frac{d}{c}$$

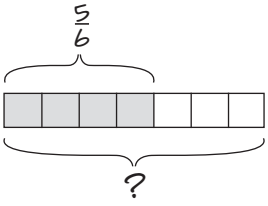
invert and multiply!

Note: $\frac{3}{2}$ is the inverse or reciprocal of $\frac{2}{3}$.

This more general rule follows from partitive interpretation:

Ex 7 $\frac{5}{6} \div \frac{4}{7}$

PD: $\frac{5}{6}$ is $\frac{4}{7}$ of what?



4 units $= \frac{5}{6}$

1 unit $= \frac{5}{6} \div 4 = \frac{5}{24}$

? $= 7$ units $= \frac{5}{24} \times 7 = \frac{35}{24}$

Abstractly: $\frac{5}{6} \div \frac{4}{7} = (\frac{5}{6} \div 4) \times 7$

$= (\frac{1}{4}$ of $\frac{5}{6}) \times 7$

$= \frac{5}{6} \times \frac{1}{4} \times 7$

$= \frac{5}{6} \times \frac{7}{4} = \frac{35}{24}$.

HW Read § 6.4 and § 6.5. Do HW set 27.

Bring Text book to next class!

Sing 5A

Sing 6A