What to bring to class:
Ask students to bring PM 4A and 5A.

6.3 Multiplication of fractions

[Do not write on board, go over in textbook:]

So far: fractions are ways to measure parts

NOW, more and more, we want to think of them as numbers which can be +, −, x, ÷.

Guiding Principles:

$$\left.\begin{array}{l}\text{Commutative}\\ \text{Associative}\\ \text{distributive}\\ \text{Identity}\end{array}\right\}\ \text{"How numbers behave!"}$$

For example, we could interpret fraction multiplication as:

$$\frac{2}{7} \text{ "x" } \frac{3}{7} = \frac{6}{7}$$ (similar to how we add fractions!)

but something strange happens

$$\frac{1}{3} \text{ "x" } \frac{2}{3} = \frac{1 \times 2}{3} = \frac{2}{3}$$

would imply $\frac{1}{3} = 1$ (since by m. ident. 1 is <u>only</u> # such that $1 \cdot a = a$)

⇒ Not a good interpretation!

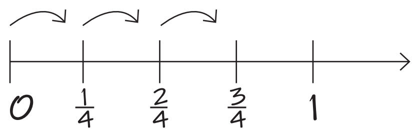
To avoid misconceptions (like the one above) teaching fractions must be done carefully!

Teaching Sequence

Case 1 whole # x fraction

old definition of x still works!

$3 \times \frac{1}{4} = 3$ groups of $\frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$



Think: $3 \times \frac{2}{5} = 3$ groups of 2 fifths = 6 fifths

$= \frac{6}{5}$

Case 2 Fraction x whole.

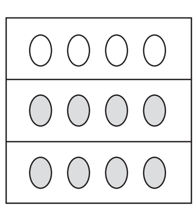
Old interpretations:

$\frac{1}{4} \times 3 = \underbrace{\frac{1}{4} \text{ groups}}_{\text{what is } \frac{1}{4} \text{ groups?}} \text{ of } 3$

$\frac{1}{4} \times 3 = \underbrace{\text{"Add 3 to itself } \frac{1}{4} \text{ times"}}_{?}$

Need a new interpretation of mult!

Ex: Note that $\frac{2}{3}$ of 12 eggs = 8 eggs.



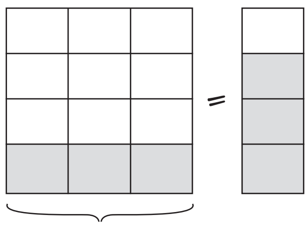
which is the same as 12 groups of $\frac{2}{3}$

$$12 \times \frac{2}{3} = \underbrace{\frac{2}{3} + \frac{2}{3} + \ldots\ldots + \frac{2}{3}}_{12} = \frac{24}{3} = 8$$

New Interpretation: $\frac{1}{4} \times 3 = \frac{1}{4}$ of 3

Try:

$\frac{1}{4}$ of



$\frac{3}{4}$'s !

3 units

see pages 44–45

<u>Note</u>: $3 \times \frac{1}{4} \overset{\text{case 1}}{=} \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} = \frac{1}{4}$ of $3 \overset{\text{case 2}}{=} \frac{1}{4} \times 3$ <u>Commutative!</u>

[SAY: By making this interpretation of fraction mult. we see that the commutative property still holds. If it <u>didn't</u>, then fractions wouldn't <u>behave</u> like numbers should!]

<u>Case 3</u> fraction x fraction

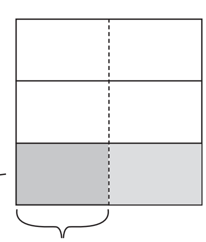
Note: $\frac{1}{2} \times \frac{8}{9} = \underbrace{\frac{1}{2} \text{ of } 8}_{\text{reduce to case 2.}}$ ninths $= 4$ ninths $= \frac{4}{9}$

Start easy: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{2}$ of $\frac{1}{3}$

new interpretation (case 2)

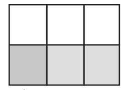
$= \frac{1}{6}$

<u>Model</u>:

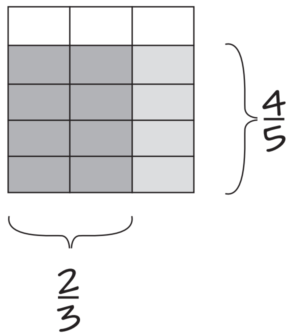


$\frac{1}{2}$ of $\frac{1}{3}$

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$



<u>Ex</u> [Students do] Model $\frac{2}{3} \times \frac{4}{5}$



$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

numerators multiply like whole #'s: 2 x 4

new fractional unit found by multiplying denominators: 3 x 5

(See pages 49–52 pf PM 5A)

The model motivates the abstract notation:

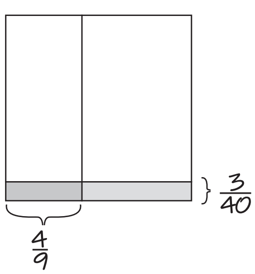
$\boxed{\text{Fraction rule 4: } \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}}$

<u>Word Problems</u>

skip if not enough time

Mr. Jackson brought $\frac{3}{40}$'s of a wood pile into the house to use in a fire. He used only $\frac{4}{9}$'s of what he brough in. How much of the wood pile did he actually use?

<u>T.S.</u>



$\frac{4}{9}$ of $\frac{3}{40} = \frac{4}{9} \times \frac{3}{40}$

$= \frac{\cancel{4} \times \cancel{3}}{{}_{3}\cancel{9} \times \cancel{40}_{10}} = \frac{1}{30}$

He used $\frac{1}{30}$ of the wood pile.

Division (Reviews of measurement and Partitive)

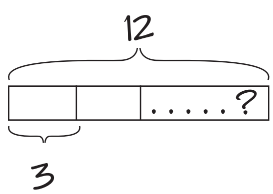
Measurement and partitive interpretations still work for fractions!

[SAY: In fact, we need them to make sense of the "invert and multiply" rule.]

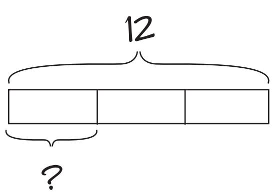
[This can't be skipped:]

[You can follow in book if out of time]

Measurement division $\quad$ $12 \div 3$ means "12 is how many 3's ?"



Partitive division $\quad$ $12 \div 3$ means "12 is 3 of what?"



In fractions, we use the same interpretations, but the digrams might be different.

Ex 1 (Measurement) If a road is created at $\frac{1}{2}$ miles per week, how many weeks are needed to build $1\frac{3}{4}$ miles?

SAY:

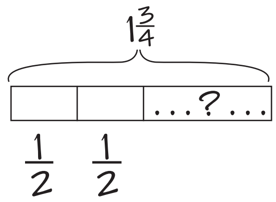
Notes: · Answer $1\frac{3}{4} \div \frac{1}{2}$.

· Liping MA study: Only 43% of U.S. teachers correctly found $1\frac{3}{4} \div \frac{1}{2}$.

· Same study: only one U.S. teacher was able to make up a word problems like ex 1.

Interpretive question: $1\frac{3}{4}$ is how many $\frac{1}{2}$'s? ← get students to come up with

Diagram:



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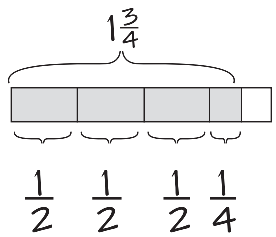


$\frac{1}{2}$ mile = 1 week

$\frac{1}{4}$ mile = $\frac{1}{2}$ week

$1\frac{3}{4}$ mile = 3 weeks + $\frac{1}{2}$ week

= $3\frac{1}{2}$ weeks

It will take $3\frac{1}{2}$ weeks.

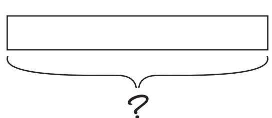
<u>Ex 2 (Partative)</u> If $\frac{1}{2}$ of a jump rope is $1\frac{3}{4}$ meters, what is the length of the rope?

interpretive question: $1\frac{3}{4}$ is $\frac{1}{2}$ of what?

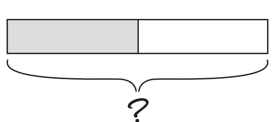
$1\frac{3}{4} \div \frac{1}{2}$.

Steps for writing down the model: [page 148 in Text book]

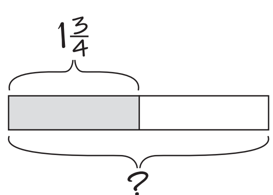
(1) Draw the "what bar" and label w/ a ?



(2) Find $\frac{1}{2}$ of it

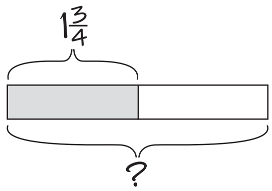


(3) Label that portion by $1\frac{3}{4}$



[SAY: $1\frac{3}{4}$ is $\frac{1}{2}$ of what? Point to each piece as you go.]

<u>T.S.</u>



1 unit = $1\frac{3}{4}$

2 units = $3\frac{1}{2}$

The rope is $3\frac{1}{2}$ meters long.

Note: $1\frac{3}{4} \div \frac{1}{2} = ? = 1\frac{3}{4} \times 2 = 3\frac{1}{2}$

[SAY: 1st indication we must "invert and multiply"]

$\boxed{\text{HW}}$ Read § 6.3 very carefully. Then do HW set 26.

Bring 5A & 6A to next class.