What to bring to class:
Ask students to bring PM 4A and 5A.

5.5 GCF and LCM.

The greatest common factor is its own definition, i.e., list all of the common factors of 2 numbers and take the largest.

Ex GCF of 36 and 84

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
Factors of 84: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

GCF

GCF (36, 84) = 12

Easier way: Prime factor both numbers. Then pair common factors.

$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$
$$84 = \underbrace{2 \cdot 2 \cdot 3}_{\text{Biggest \# which divides both.}} \cdot 7$$

$= 2 \cdot 2 \cdot 3 = 12.$

The Least common Multiple (LCM) is also its own definition.

1st: Find common multiples.
2nd: Take the smallest.

Ex Find LCM of 16, 24. (Students do)

Multiples of 16: 16, 32, 48, 64, 80, 96, 112, . . .
Multiples of 24: 24, 48, 72, 96, 120, 144
smallest!

LCM (16, 24) = 48.

Easier way:

#1 Prime factorization
#2 Pair prime factors
#3 Multiply pairs with extra primes

$$\left.\begin{matrix} 16 = \boxed{2}\cdot\boxed{2}\cdot\boxed{2}\cdot 2 \\ 24 = \underbrace{\boxed{2}\cdot\boxed{2}\cdot\boxed{2}}_{\text{pairs}}\cdot 3 \end{matrix}\right\} \text{extra's}$$

$$\text{LCM}(16, 24) = \underbrace{2 \cdot 2 \cdot 2}_{\text{pairs}} \cdot \underbrace{2 \cdot 3}_{\text{extra}} = 48$$

<u>Note</u>: $\text{GCF}(a, b) \leq \min(a, b)$

$\text{LCM}(a, b) \geq \max(a, b)$

In fact, one can define GCF and LCM in terms of prime factorization.

Prime factorizations of $a = p_1^{r_1} \cdot p_2^{r_2} \cdots\cdots p_k^{r_k}$

$b = p_1^{s_1} \cdot p_2^{s_2} \cdots\cdots p_k^{s_k}$

[SAY! P's all different from each other, but same for a and b.]

Then $\text{GCF}(a, b) = p_1^{\min(r_1, s_1)} \cdot p_2^{\min(r_2, s_2)} \cdots\cdots p_k^{\min(r_k, s_k)}$

$\text{LCM}(a, b) = p_1^{\max(r_1, s_1)} \cdot p_2^{\max(r_2, s_2)} \cdots\cdots p_k^{\max(r_k, s_k)}$

<u>Ex</u> Find GCF & LCM of 36 and 84

$36 = 2^2 \cdot 3^2 \cdot 7^0$

$84 = 2^2 \cdot 3^1 \cdot 7^1$

means we put $7^0 = 1$ to get same # of primes

$\text{GCF}(36, 84) = 2^{\min(2,2)} \cdot 3^{\min(2,1)} \cdot 7^{\min(0,1)}$

$= 2^2 \cdot 3^1 \cdot 7^0 = 4 \cdot 3 = 12$ ✓

$\text{LCM}(36, 84) = 2^{\max(2,2)} \cdot 3^{\max(2,1)} \cdot 7^{\max(0,1)}$

$= 2^2 \cdot 3^2 \cdot 7^1 = 4 \cdot 9 \cdot 7 = 9 \cdot 28 = 280 - 28 = 252$

HW Problem in 5b in HW set 22:

Prove that GCF $(a, b) \cdot$ LCM $(a, b) = a \cdot b$.

Note that $\min(r_1, s_1) + \max(r_1, s_1) = r_1 + s_1$. (*)

$$GCF(a,b) \cdot LCM(a,b) \overset{\text{def.}}{=} P_1^{\min(r_1,s_1)} \cdot P_2^{\min(r_2,s_2)} \cdots P_k^{\min(r_k,s_k)} \cdot P_1^{\max(r_1,s_1)} \cdots P_k^{\max(r_k,s_k)}$$

combine

$$= P_1^{\min(r_1,s_1)+\max(r_1,s_1)} \cdot P_2^{\min(r_2,s_2)+\max(r_2,s_2)} \cdot \cdots P_k^{\min(r_k,s_k)+\max(r_k,s_k)}$$

Any Order Power Rule 1.

$$= P_1^{r_1+s_1} P_2^{r_2+s_2} \cdots P_k^{r_k+s_k} \qquad \text{by } (*)$$

$$= (P_1^{r_1} \cdot P_2^{r_2} \cdots P_k^{r_k})(P_1^{s_1} \cdot P_2^{s_2} \cdots P_k^{s_k}) \qquad \text{Power Rule 1. Any Order}$$

$$= a \cdot b$$

HW Problem in 5c in HW set 22:

$16 = 2^4 \cdot 3^0 \cdot 17^0$

$102 = 2^1 \cdot 3^1 \cdot 17^1$

GCF $(16, 102) = 2^1 \cdot 3^0 \cdot 17^0 = 2$.

$2 \cdot$ LCM $(16, 102) = 16 \cdot 102 = 1600 + 32 = 1632$

$\boxed{\text{LCM}(16, 102) = 816.}$

Note: LCM $(16, 102) = 2^4 \cdot 3^1 \cdot 17^1 = 16 \cdot 51 = 800 + 16$

$= 816$.

$\boxed{\text{HW}}$ Read § 5.5 Do HW set 22.

Give quiz or go over HW or practice Mental Math.

[If you want, you can show
GCF (a, a + b) = GCF (a, b)
and then go on to prove the Euclidean Algorithm.]

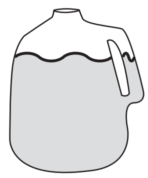
Caution – This lecture follows the book <u>EXACTLY</u>
be careful about boring students

<u>Lecture 25 – Fraction Basics</u>

Instructor – photocopy
Prim Math 2B pgs 52–57
Prim Math 3B pgs 51–62
as handout for today's class.
Students Bring PM 4A to class

Fractions used when there is a <u>standard unit</u> but we want to measure using (usually) smaller units called the <u>fractional unit</u>

<u>Ex</u> 4 quarts = 1 gallon
standard unit: gallon
fraction unit: quart
3 quarts = ?



<u>Notation</u> $\frac{3}{4}\ \underbrace{\text{gallon.}}_{\text{standard unit.}}$

<u>numerator</u> = # of fractional units
<u>denominator</u> specifies the fractional unit; it is the number of fractional units in the standard unit.

[Say: Fractional unit usually doesn't have its own name (like "quart" above) – It is defined by the denominator.]

<u>Notes</u>

(1) <u>Must</u> know the standard unit (I have $\frac{3}{4}$ water doesn't make sense)