What to bring to class:
Ask students to bring PM 4A and 5A.

5.4 More Primes

[Say: In todays class we put together the divisibility test with the Fundamental Theorem of Arithmetic (FTA)]

Test Primality

To test if a number n is composite or prime:

1st: Need only check for prime factors.

If n is composite, $n = P_1 P_2 \dots P_k$ (by FTA)

$= P_1 \cdot$ (some whole #)

$\Rightarrow$ n has prime factor.

[Say: This is obvious, if n is divisible by 6, then it must be divisible by 2 and 3.]

2nd: Need only check primes up to $\sqrt{n}$.

Ex Factors of 36: 1 2 3 4 6 9 12 18 36

(paired: 1–36, 2–18, 3–12, 4–9; 2, 3 = prime factors of 36; $6 = \sqrt{36}$)

$2 =$ smallest P.F. $\leq \sqrt{36}$

Ex What is 13 x 17 ? 221 ← MM: $15^2 - 2^2$

Factors of 221: 1 13 17 221

(paired: 1–221, 13–17)

$13 =$ smallest P.F. $\leq \sqrt{221} \approx 15$

<u>Guess:</u> $1 < \text{smallest P.F.} \le \sqrt{n}$

<u>Check:</u> $n = P_1 \cdot P_2 \cdot P_3 \cdots P_k$ by FTA

Assume P_1 is smallest P.F.

Then $n = P_1 \cdot P_2 \cdots P_k \ge P_1 \cdot P_2 \ge P_1 \cdot P_1$

$\Rightarrow n \ge P_1^2 \Rightarrow \sqrt{n} \ge P_1$ ✓

$\Rightarrow$ At least one factor of a composite # must be $\le \sqrt{n}$. If not, n is prime.

<u>Primality Test:</u> To test if a number n is prime, one need only search for prime factors p of n with $p \le \sqrt{n}$.

<u>Ex</u> Is 163 prime? $\sqrt{163} \le \sqrt{169} = 13$ (Estimate)

~~2~~ ~~3~~ ~~5~~ ~~7~~ ~~11~~ ~~13~~

- 2, 3, 5: Div. test
- 7: 140, 23, <u>No</u>
- 11: 110, 53, <u>No</u>
- 13: 169 − 6, NO

[Yes 163 is prime]

<u>Ex</u> Is 1001 prime? $\sqrt{1001} \le \sqrt{1032} = \sqrt{2^{10}}$

$= 2^5 = 32$

~~2~~ ~~3~~ ~~5~~ (7) 11 13 17 19 23 29 31

- 2, 3, 5: D.T.
- 7: 700 ✓, 280 ✓, 21 ✓ (21: Division Lemma.)

No. 1001 is not prime!

The number of primes.

Have students do the following exercise: [3 – 4 min]

Create a handout (using Mathematica, etc.) of all primes up to 2000. Ask them to count the # of primes in each of the intervals: 0 – 250, 251 – 500, 501 – 750, 751 – 1000, 1001 – 1250, 1251 – 1500, 1501 – 1750, 1751 – 2000.

Have them generate this table:

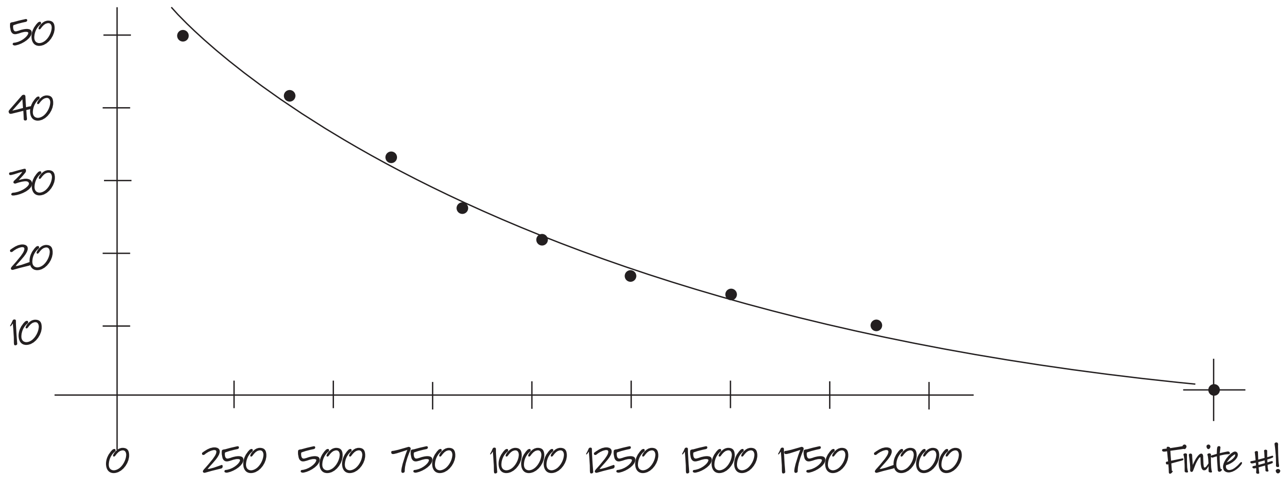
Interval	# of primes
0 – 250	50
251 – 500	42
501 – 750	37
751 – 1000	36
1001 – 1250	36
1251 – 1500	35
1501 – 1750	33
1751 – 2000	31

Ask:

- Is the # of primes increasing or decreasing?
- Make a conjecture about the # of primes.

The idea, of course, is to get them to say that there are a finite # of primes.

Then draw:



[Say: Furthermore, HW 4 in todays HW shows that for any # n, there is n consecutive #'s which are not prime. There seems to be a lot of evidence for a finite number of primes]

Tell students to put space here . . .

<u>Conjecture</u>: There are a finite # of primes.

If so, list them in order:

2, 3, 5, 7, 11, ... P ← greatest prime.

and consider $N = (2 \cdot 3 \cdot 5 \; \ldots\ldots \; P) + 1$

By FTA, since $N > 1$ it is prime or a product of primes. But it can't be prime since $N > P$. ← largest prime

Since it is a product of primes, some prime $p > 1$ in the list 2, 3, . . . , P must divide N.

Since 2 · 3 . . . . P and N are div by P, the division lemma ⇒ 1 is div. by P.

Contradiction because $p > 1$.

[Say: Our conjecture must be wrong! The opposite must be true]

There are an infinite # of primes.

[Now, go back and fill in "Theorem [Euclid] : There are an . . . . . ," before conj, erase "conjecture:" and replace with "Proof : Suppose," then put box □.

This type of proof is called "Proof by contradiction." It is useful in teaching as well:

[Say:

Jimmy states "$(a + b)^2 = a^2 + b^2$"

Tell Jimmy, if this is true, then

$(6 + 7)^2 = 6^2 + 7^2 = 36 + 49 = 85$

But $\| \quad$ +1

$13^2 = 169$ ← Contradiction!

Something must be wrong with your formula!]

HW

Read § 5.4 Do HW set 21.

List of primes up to 2000

2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113	127	131	137	13	149	151	157	163	167	173
179	181	191	193	197	199	211	223	227	229	233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349	353	359	367	373	379	383	389	397	401	409
419	421	431	433	439	443	449	457	461	463	467	479	487	491	499	503	509	521	523	541
547	557	563	569	571	577	587	593	599	601	607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733	739	743	751	757	761	769	773	787	797	809
811	821	823	827	829	839	853	857	859	863	877	881	883	887	907	911	919	929	937	941
947	953	967	971	977	983	991	997	1009	1013	1019	1021	1031	1033	1039	1049	1051	1061	1063	1069
1087	1091	1093	1097	1103	1109	1117	1123	1189	1151	1153	1169	1171	1181	1187	1193	1201	1213	1217	1223
1229	1231	1237	1249	1259	1277	129	1283	1289	1291	1297	1301	1303	1307	1319	1321	1327	1361	1367	1373
1381	1399	1409	1423	1427	1429	1433	1439	1447	1451	1453	1459	1471	1481	1483	1487	1489	1493	1499	1511
1523	1531	1543	1549	1553	1559	1567	1571	1579	1583	1597	1601	1607	1609	1613	1619	1621	1627	1637	1657
1663	1667	1669	1693	1697	1699	1709	1721	1723	1733	1741	1747	1753	1759	1777	1783	1787	1789	1801	1811
1823	1831	1847	1867	1867	1871	1873	1877	1879	1889	1901	1907	1913	1931	1933	1949	1951	1973	1979	1987
1993	1997	1999																	

Count the number of primes in each of these intervals:

Between	Number of primes
1 – 250	
251 – 500	
501 – 750	
751 –1000	
1001 – 1250	
1251 – 1500	
1501 – 1750	
1751 – 2000	

Circle one: The numbeer of primes is __________. (increasing/decreasing).

Make a conjecture about the possible number of primes based upon your observation above. Does it look like there are infinite number of primes, or a finite number?

Conjecture: There are __________ number of primes.