What to bring to class:
Ask students to bring PM 4A and 5A.

5.3 Factors and Primers

A <u>factorization</u> of a number n is a way of writing it as the product of 2 or more numbers:

$$n = a \times b$$

(a and b: factors)

<u>EX</u>

$12 = 3 \times 4$
$= 2 \times 6$
$= 2 \times 3 \times 2$
$= 12 \times 1$

All factors of 12

1, 2, 3, 4, 6, 12

Every whole number n has "trivial" factors 1 and n.

<u>Def</u> A whole number $n > 1$ is <u>prime</u> if its only factors are 1 and n. If it has at least one other factor it is <u>composite</u>.

[Say: 0 and 1 are neither prime nor composite]

[Do some examples of prime and composite]

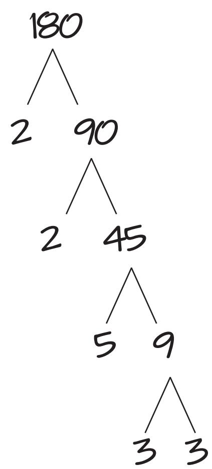
To find primes, use the "Sieve of Eratosthenes" (Era – toss – thin – e's)

[First person to estimate the Earth's circumference, tilt, size, and distance from earth to the sun and moon. – 3rd century B.C.]

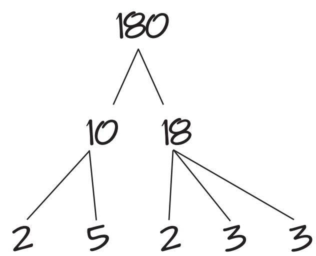
	(2)	(3)	~~4~~	(5)	~~6~~	7	~~8~~	~~9~~	~~10~~	11	~~12~~
13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~	~~21~~	~~22~~	23	~~24~~
~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~	31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~.

[Note after circling 5 and crossing out multiples, the rest are prime. Important for HW]

Repeated factoring gives a <u>factor tree</u>



– or –

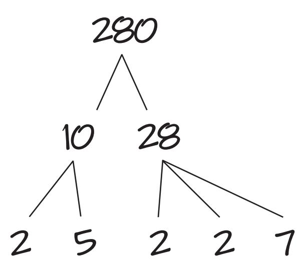


Tip: Split by large factors!

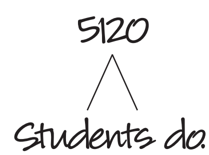
Going as far as possible yields <u>prime factorization</u> – a factorization as a product of primes.

Written $n = p_1 \cdot \ldots \cdot p_k$ (or $n = p_1$ when n is prime)

<u>Ex</u>

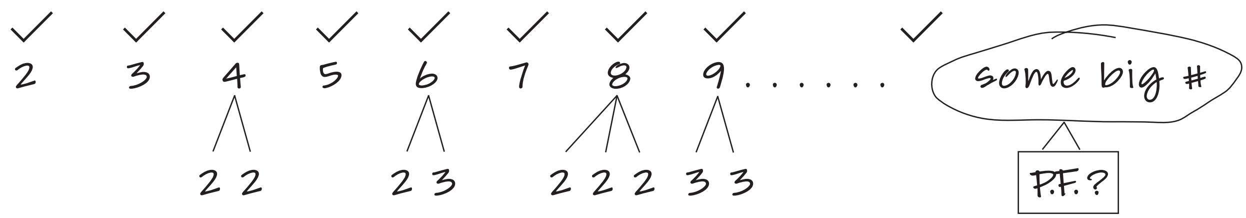


31
prime



<u>Fundamental Theorem of Arithmetic</u>. Every whole number (except 0, 1) is either prime or a product of primes. Furthermore, each whole # has only one prime factorization.

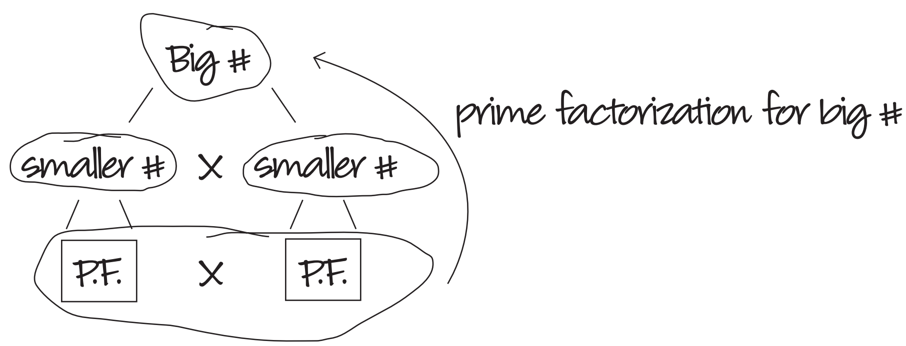
Elementary School Proof Suppose we listed every # up to some really large number and found that they all had prime factorizations.



Does that large number also have a prime factorization?

- If it is a prime – no problem, put it on our list and look at the next larger #
- If not it is the product of 2 smaller numbers – which are on our list – each have prime factorizations.

Multiplying the two prime factorizations gives a prime factorization for the large number.



In either case we can put that large # on our list and look at the next number. By the same argument, we can put that # on our list as well. [Explain why]

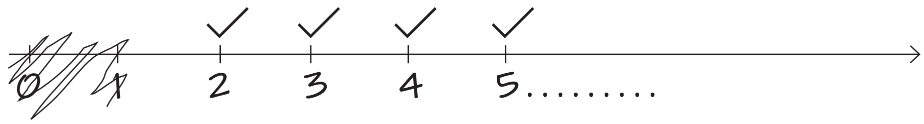
Each time we put a number on our list, it helps us to explain why we can put the next number on the list as well.

Thus our list grows until it contains every whole number!

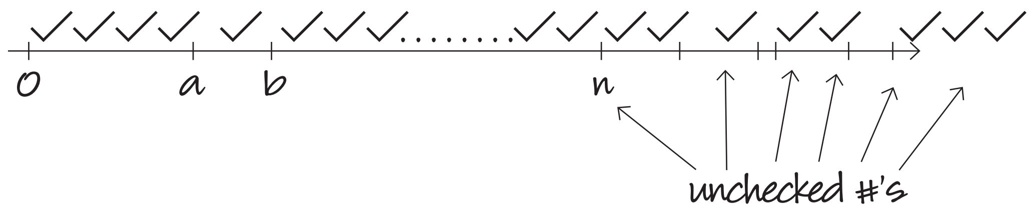
□

Proof (using different logic)

Walk along the # – line. Put a ✓ over each number which is prime or a product of primes.



Suppose some #'s can't be checked. Let n be the samllest of those numbers.



n is not prime ⟹ n is composite and must factor: $n = a \cdot b$ where a and b are smaller and checked. Each can be written as a product of primes. ⟹

$n = a \cdot b =$ (product of primes) $\cdot$ (product of primes)

n can be checked, but we said it couldn't be!

Contradiction (something is wrong)

The only possibility: Our assumption that some #'s can't be checked is wrong ⟹ all of them can be!

□

HW Read S 5.3 and do HW set 20.