What to bring to class:
Ask students to bring PM 4A and 5A.

5.2 Divisibility Test [day lecture]

All letters A, a, b, k, l, m represent whole numbers > 0.

<u>Def</u> <u>A is divisible by k</u> means k divides into A w/o remainder, i.e., there exists a quotient q such that $A = k \cdot q$.

If $A = k \cdot$ (some whole #) we can say:

* k divides A
* k is a factor of A
* k goes into A evenly
* A is divisible by k.

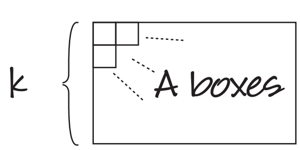
(Notation $k \mid A$ used in higher math, we do not need it however.)

[SAY: All are equivalent]

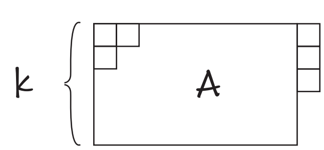
<u>Examples</u>

1. 3 divides 21
2. 75 is divisible by 5
3. 16 goes into 1024 (q = how many times $= 2^{10} \div 2^4 = 2^6 = 64$ times)
4. 3 is not a factor of 14

We can model "A is div. by k" by



A is div. by k



A is <u>not</u> div. by k

Ex Note 16 is div. by 4. What other #'s are?

16 + (0)	✓
16 + 1 = 17	x
16 + 2 = 18	x
16 + 3 = 19	x
16 + (4) = 20	✓
16 + 5 = 21	x
⋮	
16 + B	✓ if B is div. by 4 x if B is not div. by 4.

Statement (1): Suppose A is div. by k (Think 16). If B is divisible by k (Think 4) (2nd column) $\Longrightarrow$ (A + B) is div. by k. (3rd column)

($\Longrightarrow$ means "then" or "implies")

Now look at sums which equal 28 (which is div. by 4). When is one of the addends div. by 4, when is the other?

28 = 0 + 28	✓
28 = 1 + 27	x
28 = 2 + 26	x
28 = 3 + 25	x
28 = 4 + 24	✓
28 = 5 + 23	x
28 = 6 + 22	x
28 = 7 + 21	x
28 = 8 + 20	✓
⋮	
28 = A + B	✓ When A is, then B is too! x neither are!

(Think 0,4,8 above) (Think 28)

Statement (2): Suppose A is div. by k. If A + B is divisible by k $\Longrightarrow$ B is also div. by k.

We combine statements (1) and (2) by using $\Longleftrightarrow$:

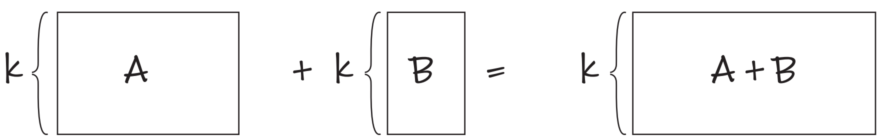
[Say: "if and only if"]

Division Lemma: Suppose A is div. by k. Then B is div. by k, $\Longleftrightarrow$ A + B is div. by k.

[Write only $\Longrightarrow$ up first, then read the last sentence backwards to get statement (2), filling in the "$\Longleftarrow$" as you read it out loud.]

[SAY: For theorems with $\Longleftrightarrow$ in the statement, we must prove both directions.]

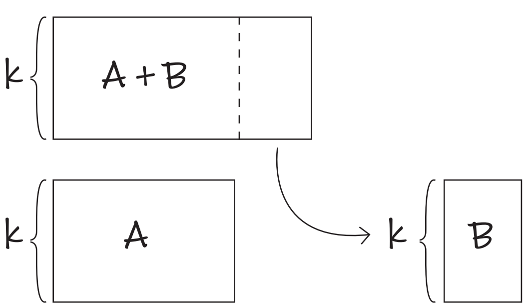
Picture proof: "$\Longrightarrow$" A is div. by k. B is div. by k $\Longrightarrow$ A + B is also



"A is div. by k" "B is div. by k" the sum must also!

□

"$\Longleftarrow$" If A and A + B are div. by k, then B is also.



B must have k rows, so it is div. by k!

□

Algebraic Proof:

A is div. by k means $A = k \cdot a$ for some a.

"$\Longrightarrow$" If B is div. by k, then $b = k \cdot b$ for some b.

Hence $(A + B) = k \cdot a + k \cdot b = k(a + b)$ ← think = "k · (some whole number)"

(substitution) (distributive property)

By definition A + B is div. by k.

"$\Longleftrightarrow$" If A + B is div. by k, then by definition $A + B = k \cdot m$ for some whole # m. Then

$$B = (A + B) - A = k \cdot m - k \cdot a$$
$$= k(m - a) = k \text{ (some whole \#)}$$

Hence B is div. by k.

[When doing the algebraic Proofs, label the columns in the picture proofs by a, b, m respectively.]

HW Read 5.2, do problems 1–5 of HW set 19.

[Day 2]

The Division is very powerful.

Ex Is 384 divisible by 6?

360 is }
24 is } $\Longrightarrow$ 384 is !
Div. Lemma

Ex Is 454 divisible by 9?

450 is }
4 isn't } $\Longrightarrow$ 454 isn't.
Div. Lemma

There are quicker ways to test for divisibility:

Divisibility Test: A number is divisible by

10	$\Longleftrightarrow$	ends with 0
5	$\Longleftrightarrow$	ends with 0, 5
2	$\Longleftrightarrow$	ends with 0, 2, 4, 6, 8
4	$\Longleftrightarrow$	last 2 digits are div. by 4
8	$\Longleftrightarrow$	last 3 digits are div. by 8
3	$\Longleftrightarrow$	sum of its digits are div. by 3
9	$\Longleftrightarrow$	sum of its digits are div. by 9
11	$\Longleftrightarrow$	if the difference between odd-position and even-position digits is div. by 11

Examples:

① is 8176 div. by 2? by 4? by 8?

② is 11,265 div. by 3? by 9?

③ Which divides 84,570?

(2) (3) ~~4~~ (5) ~~8~~ ~~9~~ (10)

④ Is 874, 256, 921, 832 div. by 3, 9?

Tip: "Cast out 9's"

~~874~~, ~~256~~, ~~921~~, ~~832~~?

8 + 2 + 2 = 12 { div. by 3, not by 9.

⑤ Is 87365 div. by 11?

$(8+3+5) = 16$
$(7+6) = \underline{-13}$
③ ⟵ not div. by 11
⟹ 87365 not div. by 11.
(but 873<u>9</u>5 is. Why?)

Why are div. test true? Use the Division Lemma.

[Write the following template on the board w/o the blanks, and doing the test for 2 at the same time.]

[Template: Use lots of space!]

Proof of the test for $\underline{2}$:

By place value, any number n can be written

$n = \underline{10a + b}$

This # is $\underline{2\,(5a)}$, so it is div. by $\underline{2}$.

This is the test #, i.e, $\underline{\text{it is the last digit}}$.

By the Division Lemma,

n is div. by $\underline{2}$ ⟺ $\underline{b \text{ is div. by } 2}$
⟺ $\underline{b = 0, 2, 4, 6, 8}$ □

[If you have to, give numerical examples along the way. For ex, write:
$124 = \underbrace{12}_{a} \times 10 + \underbrace{4}_{b}$, and so forth.]

[Tell them that all other proofs are similar.
1. Use place value to break # into the test case and a # which is div. by test #, and
2. Apply Division Lemma.]

[Then do repeatedly erasing the blanks and filling in with new proof.]

Fast: Erase and fill in proofs for 5 and 10 – tell students not to copy them down.

Slow: Give time for them to recopy template, Fill in proof for test for 4.

$$n = 100a + b$$

$100a$ ← $4(25a)$; b ← last 2 digits < 100

Fast: Same for 8

Slow: Give time to recopy template. Fill in proof for 3 (using 3 digit # abc)

$$\begin{aligned} n &= 100a + 10b + c \\ &= (99a + a) + (9b + b) + c \\ &= (99a + 9b) + (a + b + c) \end{aligned}$$

$(99a + 9b)$ ← $9(11a + b)$; $(a + b + c)$ ← sum of digits

Fast: Same for 9.

Slow (very slow)

Test for 11, n with digits a b c d

$$\begin{aligned} n &= 1000a + 100b + 10c + d \quad \text{P.V.} \\ &= (1001a - a) + (99b + b) + (11c - c) + d \\ &= (1001a + 99b + 11c) + (-a + \underline{b} - c + \underline{d}) \end{aligned}$$

$(1001a + 99b + 11c)$ ← $11(91a + 9b + c)$ is div. by 11

$(-a + b - c + d)$ ← test case $(b + d) - (a + c)$ "odd" – "even"

HW Read §5.2 again, do the rest of HW set 19 [do not turn in previous work].