

What to bring to class:
Ask students to bring PM
4A and 5A.

5.1 - Even/Odds - Intro to Proofs

Ask students to write down their definition of "even number."
Explain why even + even = even using their definitions.

Compile list of defs

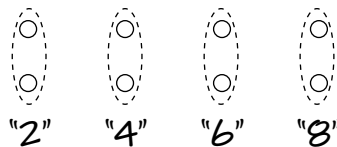
4 different definitions of even number:

- a) a number which occurs by skip-counting by 2. (good to introduce)
- b) an even number of objects can be paired up with no remainder
(visual, used in pic proofs)
- c) a number which is twice a whole number (can be represented as $2n$ for some whole # n) (general, used in alg proofs)
- d) a number whose last digit is 0, 2, 4, 6, or 8.

say: to adults all 4 seen the same & are part of our notion of "even." But they are different (d) depends on decimal notation). Children must learn all 4 one at a time & link them. (Teaching & math exercise)
Ex: 3574 even? check with different def's. Which is easier?

Links:

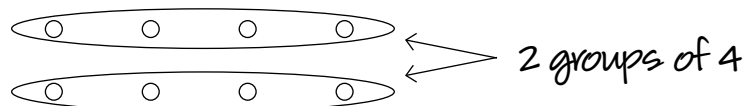
(a) \implies (b)



links def. 1 to def. 2

*add new pair at each step
*none left \implies even
1 left \implies odd

(b) \implies (c)



links def. 2 to def. 3

(a) \implies (a) count w/ no skips

1 2 3 4

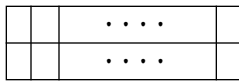
doubles \implies skip counting

double each #

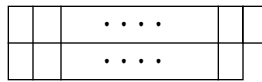
$\downarrow \downarrow \downarrow \downarrow$
2 4 6 8

will do (d) later - see PM 4A pg25 for early connection for $a \iff d$.

Array Pictures: (for clearer/simpler pics)



even



← meaning of dots.
not a pic of one
odd number, it is
a schematic for all
odds.

Simple Proofs:

Def: A proof is a detailed explanation of why a mathematical fact is true

* follow from basic rules of reasoning ← same goal as teaching

* communicate - everything in math makes sense.

* Theorems - are proven fact

* Lemmas - simple theorems which are used repeatedly.

Proofs start with "Proof:" and end with " \square "

Ask students about their experiences in high school.

Remember proofs = explanations.

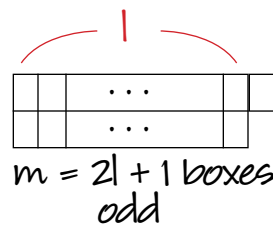
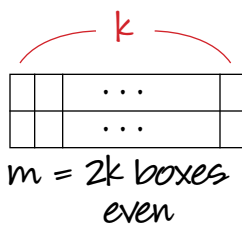
Proofs in Elementary school - informal, done with models, pics, numbers.

Theorem 1: The sum of two even number is even.

Proof: (picture)



explain
start w/
end w/



Proof (algebraic) Two even numbers can be written as

→ ask: def (c)

$2k$ and $2l$. Then

$$2k + 2l = 2(k + l)$$

Distributive Property (ask)

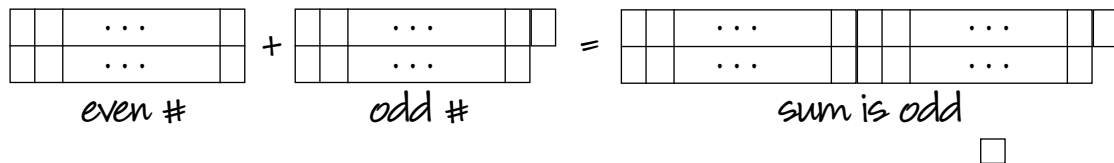
is even by def (3)

□

Theorem 2: "even + odd = odd"

The sum of an even number and an odd number is odd.

Proof: (picture)



Proof: (Algebraic)

Given an even number $2k$ and an odd number $2l + 1$

$$2k + (2l + 1) = (2k + 2l) + 1 \quad \text{Associative Property}$$

$$= 2(k + l) + 1 \quad \text{Distributive Property}$$

is odd.

□

* These are real proofs!

Say: Reasoning is clear. Note: Pictures become awkward when they involve large numbers or arbitrary numbers.

HW - Read 5.1 Do HW Set #19.