What to bring to class:
Ask students to bring PM 4A and 5A.

4.3 – Exponents

Write $2^n = \underbrace{2 \cdot 2 \cdot 2 \cdots 2}_{n \text{ times}}$

say: Just notation!

<u>Ex</u>: $2^5 = 32$
$2^8 = 256$
$2^{10} = 1024$

<u>Def</u>: $a^n = \underbrace{a \cdot a \cdots a}_{n}$ for a, n positive whole numbers.

exponent

base

<u>Consequences of Definition</u>:

<u>Ex</u>: $2^3\, 2^7 = \underbrace{(2 \cdot 2 \cdot 2)}^{3} \cdot \underbrace{(2 \cdot 2 \cdot 2 \cdots 2)}^{7}$
= product of 10 2's
$= 2^{10}$

<u>Rule 1</u>: $a^n \cdot a^m = a^{m+n}$

say: Just counting the number of factors

<u>Mental Math</u>: $32 \times 64 = 2^5 \cdot 2^6 = 2^{11} = 2^{10} \cdot 2 = 1024 \cdot 2 = 2048$

<u>Ex</u>: A germ cell divides every hour. how many cells in 36 hours?

$2^{36} = 2^{10} \cdot 2^{10} \cdot 2^{10} \cdot 2^6 = 1024 \cdot 1024 \cdot 1024 \cdot 64 \approx 7$ billion compensation!

Ex: $2^5 \div 2^2 = \dfrac{2\cdot 2\cdot 2\cdot \cancel{2}\cdot \cancel{2}}{\cancel{2}\cdot \cancel{2}} = 2\cdot 2\cdot 2 = 2^3$

$$2^{19} \div 2^{13} = \frac{\overbrace{2\cdot 2\cdots 2}^{19}}{\underbrace{2\cdot 2\cdots 2}_{13}} = \underbrace{2\cdot 2\cdots 2}_{6} = 2^6$$

Rule 2	$a^m \div a^n = a^{m-n}$	when $a \neq 0$, $m \geq n$

In general:

$a^m \div a^n = \dfrac{a^m}{a^n}$ fraction notation

$= \dfrac{\overbrace{\cancel{a\cdots a}}^{n}\cdot \overbrace{a\cdots a}^{m-n}}{\underbrace{\cancel{a\cdots a}}_{n}}$ def

$= \underbrace{a\cdots a}_{m-n}$ simplify

$= a^{m-n}$ def

Teaching Aside: What is $x^2 + x^3$?

Common Mistake: x^5.

In fact, doesn't simplify. Exponential rules apply only when $\times$ and $\div$ (no $+$, $-$) and are just counting factors.

Ex: $(2^3)^4 = \underbrace{2^3\, 2^3\, 2^3\, 2^3}_{12\ 2\text{'s}} = 2^{12} = 4096$

$\underline{\text{Rule 3}}$ $\quad (a^m)^n = a^{mn} \quad$ for $a \neq 0 \quad m, n$

$$(a^m)^n = \underbrace{a^m \cdot a^m \cdots a^m}_{n\text{-times}} \qquad \text{Def}$$

$$= a^{\overbrace{m+m+m+\ldots+m}^{n}} \qquad \text{Rule 1}$$

$$= a^{mn} \qquad \text{Def of Mult.}$$

$\underline{\text{Ex}}$: $2^3 \cdot 5^3 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$ $\quad$ def

$= 2 \cdot 5 \cdot 2 \cdot 2 \cdot 5 \cdot 5$ $\quad$ CP

$= 2 \cdot 5 \cdot 2 \cdot 5 \cdot 2 \cdot 5$ $\quad$ CP

$= (2 \cdot 5)(2 \cdot 5)(2 \cdot 5)$ $\quad$ Assoc.

$= (2\ 5)^3$

$\underline{\text{Rule 4}}$ $\quad a^m b^m = (ab)^m$

"When two bases have same exponent, we can form pairs"

$$a^m b^m = (\underbrace{a \cdots a}_{m})(\underbrace{b \cdots b}_{m}) \qquad \text{def}$$

$$= \underbrace{(ab)(ab)\cdots(ab)}_{m} \qquad \text{Any - order}$$

$$= (ab)^m \qquad \text{def}$$

These 4 rules are statements about <u>numbers</u>.

$\underline{\text{Ex}}$: (similar to HW)

Simplify $\quad \dfrac{2^5 \cdot 6^2 \cdot 18^2}{3^4 \cdot 4^2}$

Idea: factor into only 2's & 3's, then count up total of 2's & 3's.

$$= \frac{2^5\cdot(2\cdot3)^2\cdot(2\cdot3\cdot3)^2}{3^4\cdot(2\cdot2)^2} = \frac{2^5\cdot\cancel{2^2}\cdot3^2\cdot\cancel{2^2}\cdot\cancel{(3^2)^2}}{\cancel{3^4}\cdot\cancel{2^4}} = 2^5\cdot3^2 = 32\cdot9$$

$$= 320 - 32$$

$$= 288$$

What about 5^0?

Pattern:

$$\begin{aligned} 5^3 &= 125 \\ &\downarrow \div 5 \\ 5^2 &= 25 \\ &\downarrow \div 5 \\ 5^1 &= 5 \\ &\downarrow \div 5 \\ 5^0 &= \_ \end{aligned}$$

Guess $5^0 = 1$. Is this consistent with Rules 1 – 4?

① $5^0\cdot5^m = 5^{0+m} = 5^m$ only if $5^0 = 1$ ✓

② $5^0 = 5^{m-m} = \frac{5^m}{5^m} = 1$ ✓

③ $(5^0)^m = 5^{0\cdot m} = 5^0$ 1 is only # so that $\underbrace{1\cdot1\cdots1}_{m} = 1 \Rightarrow 5^0 = 1$.

④ $(5\cdot7)^0 = 5^0\cdot7^0 = 1\cdot1 = 1$

Completely consistent! So ok to define $a^0 = 1$ for all $a \neq 0$.

What about 0^0?

Patterns:

$$\begin{aligned} 3^0 &= 1 & \qquad 0^3 &= 0 \\ 2^0 &= 1 & 0^2 &= 0 \\ 1^0 &= 1 & 0^1 &= 0 \\ 0^0 &= \_ & 0^0 &= \_ \end{aligned}$$

*Suggests can't define 0^0 consistently.

*If we want $a^{m-n} = a^m \div a^n$ then $0^0 = 0^{1-1} = 0 \div 0$ undefined!

$\boxed{\text{HW}}$ Read 4.3

Do HW #18