

EXIT WITH EXPERTISE

Are You Qualified To Teach Elementary School Math?

We offer this test to suggest the nature and rigor of the mathematics preparation that should be offered to elementary teacher candidates so that they can excel in classroom instruction. Every prospective and practicing elementary teacher should be able to solve these problems without use of a calculator. Anyone who can't solve most of them might want to consult the elementary mathematics textbooks that we rated as best for teacher preparation (see page 76).

The problems are divided into two sections:

- Section I: Constructed response (11 problems)
- Section II: Selected response (20 problems)

Some problems in Section II are "multiple choice," but the majority have answers that are selected using a method that although unfamiliar to many Americans, has many advantages for use in machine scoreable testing. It involves selecting one or more digits that comprise the answer from an array of digits on the answer sheet. Several examples follow:

Sample problem: $2 + 2 = [1]$. ← "[1]" means that the answer should be circled on row 1 of the answer sheet.

Answer sheet: [1]: 0 1 2 3 ④ 5 6 7 8 9 NA*

Sample problem: $1/2 + 1/3 = [2]/[3]$ ← "[2]/[3]" means that the answer is a fraction with a numerator that should be circled on row 2 of the answer sheet and a denominator that should be circled on row 3.

Answer sheet: [2]: 0 1 2 3 4 ⑤ 6 7 8 9 NA
 [3]: 0 1 2 3 4 5 ⑥ 7 8 9 NA

* NA stands for "no answer"



Section I: Constructed Response

- Let n and m be odd numbers.
 - Give a picture proof (e.g., array model) that the product nm is odd.
 - Prove algebraically that n^2 is odd.
 - Prove algebraically that when n^2 is divided by 4, the remainder is 1.
 - Prove algebraically that when n^2 is divided by 8, the remainder is 1.
 - Find an odd n such that n^2 divided by 16 leaves a remainder that is not 1.
- For each whole number n find the greatest common divisor (GCD) of $3n+1$ and $n+1$.
- Explain as you would to a 4th grade class each step in the multiplication of 32 by 14.
- Fill in the missing portions of the formal argument to justify that $(-a)(-b) = ab$.
 - $-b$ is the number such that $b+(-b) = 0$ by the definition of the _____.
 - $a(b+(-b)) = a0 =$ _____.
 - Using the distributive property, $a(b+(-b)) =$ _____.
 - Since $ab + a(-b) = 0$, $a(-b)$ must be equivalent to _____.
 - $-a(b+(-b)) = -a0 =$ _____.
 - Using the distributive property, $-a(b+(-b)) =$ _____.
 - Since $(-a)b + (-a)(-b) = -(ab) + (-a)(-b) = 0$, $(-a)(-b)$ must be equivalent to _____.
- Make up first grade problems of the following types:
 - The take-away interpretation for finding 15-7.
 - The part-whole interpretation for 26-4.
 - The comparison interpretation for 17-5.
- Write all the three-digit numbers that have one digit equal to 1, one digit equal to 2, and one digit equal to 3. Add them all up.
 - Show that the sum is evenly divisible by 37.
 - What would have happened if you had used different digits? Try the same problem with 2, 7 and 8.
 - Will the sum always be divisible by 37? Explain your answer.
- A store has a sale with a $d\%$ discount and must add a $t\%$ sales tax on any item purchased. Which – if either – would be cheaper for any purchase?
 - Get the discount first and pay the tax on the reduced amount.
 - Figure the tax on the full price and get the discount on that amount (assuming that a discount on a tax is possible). Justify your answer.
- You determine that $\frac{2}{3}$ of a gallon of blue paint and $\frac{1}{2}$ gallon of red paint will make a pleasant shade of purple. In a manner that can be understood by 5th graders, solve for the number of gallons of blue paint and red paint you will need to make 84 gallons of this purple paint.
- John's shop sells bicycles and tricycles. One day there are a total of 176 wheels and 152 pedals in the shop. Find the number of tricycles via a "verbal" argument that does not involve algebra.
- Make up a short word problem that builds the given expression in the given context. Be sure to make clear what the x represents:
 $(240-x)/50$ as the time needed to complete a trip to another city.

11. You are visiting a garden that begins at point A and ends at point D, with two posts (points B and C) between points A and D. There are two ways to walk through the garden: 1) a path that starts at point A and ends at point D, traveling along the circumference of a circle that has \overline{AD} as a diameter, and 2) a path that starts at point A and ends at point D, traveling along half of the circumferences of each of three circles, the first with diameter $\overline{DC} = c$, the second with diameter $\overline{CB} = b$, and the last with diameter $\overline{BA} = a$.



You walk the top path going from point A to point D and the other path along the three semicircles going from point D back to point A. Is one path longer than the other? Explain why.

Answer key starts on page 7.

Section II: Selected Response

Answer sheet starts on page 12.

12. Which of the following word problems express(es) the partitive meaning of division for $50 \div 10$?
- Ten equivalent bags contain a total of 50 cookies. How many cookies are in each bag?
 - There are 10 flowers in each of many bouquets. How many bouquets should you buy in order to purchase 50 flowers?
 - A book dealer pays \$2 to ship a box of books and spent \$20 yesterday to ship 50 books. Is he correct to conclude that 5 books were packed in each box?
- I only
 - II only
 - I and II
 - I and III
 - I, II and III
13. a. Find a rational number expressed as an unsimplified fraction that is midway between 3.1 and 3.11. Mark the correct digits for the fraction.
- $$\frac{[1] [2] [3] [4]}{[5] [6] [7] [8]}$$
- Which of the numbers below is an irrational number between 3.1 and 3.11?
 - $\sqrt{9.5}$
 - $\sqrt{9.7}$
 - $3.1011\overline{1}$
 - $3.1 + \sqrt{(2)/1000}$
 - None of the above
14. This problem investigates the interaction of the two operations of adding and rounding. It deals with two-digit numbers and the operation of rounding them to the nearest 10. The point is to compare the sum of the two rounded numbers with the rounding of their sum. Let the rounding rule be that 5 rounds up (i.e., 45 would round to 50).
- For what pair(s) of numbers below does the sum of the pair round to 80?
 - (33, 45)
 - (38, 47)
 - (36, 48)
 - (32, 42)

- a) I and II
- b) IV
- c) II and III
- d) I and III
- e) I

Think about adding a two-digit number with tens digit 3 to a two-digit number with tens digit 4. Suppose the ones digit of each can be any digit at random (i.e., with equal likelihood).

- b. In decimal form, what is the probability that the sum will round to 80?
[9].[10] [11]
- c. In decimal form, what is the probability that the sum will round to 70?
[12].[13] [14]

Suppose that one of the numbers rounds to 30 and one rounds to 40.

- d. In decimal form, what is the probability that the sum rounds to 70?
[15].[16] [17]
- e. In decimal form, what is the probability that the sum rounds to 80?
[18].[19] [20]

Suppose that one number rounds to 40 and one to 50.

- f. In decimal form, what is the probability that the sum rounds to 90?
[21].[22] [23]
- g. In decimal form, what is the probability that the sum rounds to 80?
[24].[25] [26]

Suppose both numbers round to 40.

- h. In decimal form, what is the probability that the sum rounds to 80?
[27].[28] [29]

15. a. Let a , b and c represent digits (i.e., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9). What condition on the three digits a , b and c is necessary and sufficient for the three-digit number abc , representing the sum $(100a+10b+c)$, to be divisible by 3?
- a) $a+b+c$ is divisible by 9
 - b) $a+b+c$ is divisible by 3
 - c) $2b+c$ is divisible by 4
 - d) $2b+c$ is divisible by 3
 - e) None of the above

- b. What condition on the digits a , b and c is necessary and sufficient for the three-digit number abc to be divisible by 4?

- a) $a+b+c$ is divisible by 4
- b) $2a+b+c$ is divisible by 4
- c) $2b+a$ is divisible by 4
- d) $2b+c$ is divisible by 4
- e) None of the above

The letters a and b are digits (i.e., 0, 1, 2, ..., 9).

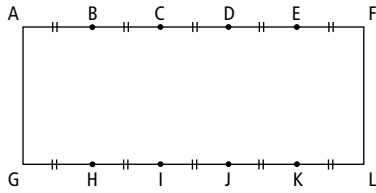
Find all choices of a for the given values of b so that $2a1181b4$ is divisible by 12, but not divisible by 9.

Note: When entering your answers below for the possible values of a , enter them in order of increasing value so that the answer you select on array #31 is more than the answer you select from array #30.

- c. If $b = 3$, a can equal:
[30], [31] or [32]
- d. If $b = 4$, a can equal
[33], [34] or [35]

16. John's shop sells bicycles and tricycles. One day there are 176 wheels and 152 pedals in the shop. How many bicycles are available for sale in John's shop that day?
[36] [37]
17. It takes one corn mill 6 minutes to grind a 50 pound bag of corn into cornmeal, while it takes a slower mill 9 minutes to grind a bag of the same size. If both mills are working at the same time, how many minutes would it take to grind 1500 pounds of corn?
[38] [39] [40]
18. Select the answer that arranges 10^{10} , 3^{20} and 2^{40} in order of size:
- a) $2^{40} < 3^{20} < 10^{10}$
 - b) $3^{20} < 2^{40} < 10^{10}$
 - c) $10^{10} < 2^{40} < 3^{20}$
 - d) $3^{20} < 10^{10} < 2^{40}$
 - e) $10^{10} < 3^{20} < 2^{40}$
19. The day you begin to store a watermelon in your basement it weighs 10 pounds and is 99% water. You forget about it for a while and when you bring it up from the basement, it is 98% water. What does it weigh in pounds?
[41].[42] [43]

20.



Let b represent the base of the rectangle and h represent its height.

A different polygon is drawn within each of three rectangles with vertices AFLG.

Polygon No. 1: A parallelogram with vertices DFIG

Polygon No. 2: A trapezoid with vertices EFJG

Polygon No. 3: A triangle with vertices ALH

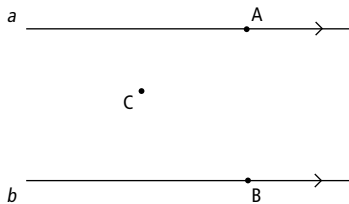
a. The area of the parallelogram is how many times the area of the triangle?

- a) 2
- b) 1.5
- c) 1
- d) 0.8
- e) 0.75

b. The area of the trapezoid is how many times the area of the triangle?

- a) 2
- b) 1.5
- c) 1
- d) 0.8
- e) 0.75

21. Lines a and b are parallel. Connect points A and C, and points B and C with line segments. The measurement of the acute angle with its vertex at point B created by \overline{CB} is 40° ; the measurement of the acute angle created by \overline{CA} with its vertex at point A is 30° . Find the measurement of $\angle ACB$.



[44] [45].[46] [47]

22. The population density of a region is the total population of the region divided by the area of the region (usually expressed in square miles or square kilometers). Asia (excluding Siberia) contains about 20% of the world's land area, but is home to about 60% of the world's people.

a. What is the ratio of the population density in Asia to the population density of the rest of the world? [48]:[49]

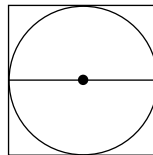
b. If the overall population density of the world is 110 people per square mile, what is the population density of Asia per square mile? [50] [51] [52]

c. What is the population density of the rest of the world per square mile? [53] [54]

23. Write the repeating decimal $0.572727\overline{2}$ as a fraction that is not reduced.

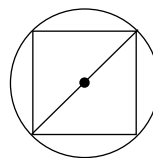
$$\frac{[55] [56] [57]}{[58] [59] [60]}$$

24. Consider a round peg with a radius r in a square hole, where the length of a side of the square hole equals the diameter of the round peg.



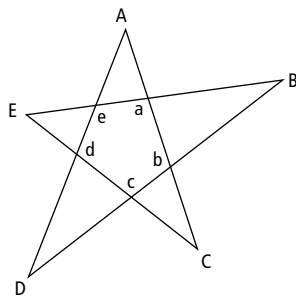
a. To an accuracy of two decimal places, what is the ratio of the area of the difference between the area of the round peg and the square hole to the area of the square hole? In other words, what is the ratio of the open space to the total area? (Use 3.14 as the approximation for π at the conclusion of your calculations.) [61].[62] [63]

Now consider a square peg in a round hole, where the diameter of the round hole equals the length of the diagonal of the square peg.



- b. To an accuracy of two decimal places, what is the ratio of the difference between the area of the round hole and the square peg to the area of the round hole? In other words, what is the ratio of the open space to the total area? (Use 3.14 as the approximation for π at the conclusion of your calculations.)
[64].[65] [66]

25. What is the sum of the measures of the angles at the vertices A, B, C, D, and E of the five-pointed star below?



[67] [68] [69] [70] [71]

26. Suppose you are looking down a road and you see a person ahead of you. You hold out your arm and sight the person with your thumb, finding that the person appears to be as tall as your thumb is long. Assume that your thumb is 2 inches long, and that the distance from your sighting eye to your thumb is 22 inches. If the person is 6 feet tall, then the person is [72] [73] feet from your eye.
27. Describe the symmetries of the square.
- [74] symmetries are reflections in lines.
 - [75] symmetries are rotations through non-zero angles.

One kind of symmetry transformation of the square is a reflection in the perpendicular bisector of two opposite sides. Another kind is reflection in the line joining two opposite vertices.

- Describe the symmetry transformation that results from doing a transformation of the first type, followed by a transformation of the second type.

- Reflection through one of the two perpendicular bisectors
- Rotation through 90°
- Rotation through 180°
- Reflection through the other diagonal
- None of the above

28. Draw a parallelogram that has a base of 1 unit, an area of 1 unit squared, and a perimeter of $36/5$ units. The coordinates of the remaining two vertices of this parallelogram in the first quadrant with the vertices of the base being (0,0) and (1,0) are

a. $x = \frac{[76] [77]}{[78]}$ $y = [79]$

and

b. $x = \frac{[80] [81]}{[82]}$ $y = [83]$

29. From a point O that is located between points A and B on \overleftrightarrow{AB} , draw \overrightarrow{OC} where C is a point not on \overleftrightarrow{AB} . Let \overrightarrow{OM} bisect $\angle AOC$ and let \overrightarrow{ON} bisect $\angle BOC$. To an accuracy of two decimal places, what is the measure of $\angle MON$ in degrees?

[84] [85] [86].[87] [88]

30. A cylindrical container can hold two quarts of water. If its radius and height are both doubled, it can hold [89] [90].[91] [92] quarts of water.
31. When rolling two dice, the probability of having the two faces add to 7 is $\frac{[93]}{[94] [95]}$ in non-reduced form, which is $\frac{[96]}{[97] [98]}$ greater than the probability of having the two faces add to 6.

Answer key starts on page 9.

Answer Key

EXIT WITH EXPERTISE Are You Qualified To Teach Elementary School Math?

Section I: Constructed Response

1. a. As a generic example consider:

○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○

There are an odd number of rows, so pair them going down to get an even number with just the last row left over. This has an odd number of circles, so pair them from the left, leaving the one in the lower right hand spot unpaired, showing that the remainder when divided by 2 is 1 and the product is odd.

- b. If n is an odd number, it can be represented as $2w+1$, where w represents a whole number (0, 1, 2, ...). $n^2 = (2w+1)^2 = 4w^2+4w+1 = 2(2w^2+2w) + 1$, so n^2 is odd.

Helpful reminder for (c) and (d) below: In division with a remainder, when dividing by a number k , the result is a whole number and a remainder, with the remainder less than k (and greater than or equal to 0).

- c. $n^2 = (2w+1)^2 = 4w^2+4w+1 = 4(w^2+w) + 1$.
Since w^2+w is a whole number and 1 is less than 4, the remainder when dividing by 4 is 1.
- d. $n^2 = (2w+1)^2 = 4w^2+4w+1 = 8[(w^2+w)/2] + 1$.
The expression $w^2+w = w(w+1)$, and either w or $w+1$ is even, so $(w^2+w)/2$ is a whole number. Thus the remainder when dividing by 8 is 1.
- e. Many odd numbers leave a remainder that is not 1, when their square is divided by 16. The number 3 is the least odd number that satisfies this condition: $3^2 = 9$, and when this is divided by 16 the quotient is 0 and the remainder is 9.

2. If the number f divides $n+1$ and $3n+1$, then it also divides $3(n+1) = 3n+3$, and $3n+1$. So f divides $(3n+3) - (3n+1) = 2$. Therefore $f = 1$ or 2.

If n is even, then $n+1$ is odd and is not divisible by 2 and the GCD is 1.

If n is odd, then $3n$ is also odd and $n+1$ and $3n+1$ are both even and the GCD is 2.

3. Multiplying 32 by 14 is the same as multiplying $32(4+10)$.

The first product line is found by multiplying 32 by 4. This process can be broken down as well as it is the same as multiplying $4(2+30)$. We multiply 2 ones by 4 ones and get a product of 8. We then multiply 30 by 4 and get a product of 120. The sum of 8 and 120 is 128.

The second product line is found by multiplying 32 by 10. The process can be broken down: it is the same as multiplying $10(2+30)$. We multiply 10 by 2 for a product of 20, and 10 by 30 for a product of 300. The sum of 20 and 300 is 320.

The sum of the two products of 128 and 320 is 448.

$$\begin{array}{r} 32 \\ \times 14 \\ \hline 8 \qquad 4 \times 2 \\ 120 \qquad 4 \times 30 \\ \hline 300 \qquad 10 \times 30 \\ 448 \end{array}$$
$$\begin{array}{r} 32 \\ \times 14 \\ \hline 128 \qquad 4 \times 32 \\ 320 \qquad 10 \times 32 \\ \hline 448 \end{array}$$



4. a. additive inverse
b. 0
c. $ab+a(-b)$
d. $-(ab)$
e. 0
f. $(-a)b+(-a)(-b)$
g. ab
5. a. How many of 15 socks are left in a laundry basket if 7 are removed?
b. Four children in a class return to an empty classroom. If there are 26 students in the class, how many more need to return to the classroom before everyone is present?
c. Seventeen dogs stayed in a pet hospital Monday night. Five dogs stayed Tuesday night. How many more dogs stayed at the hospital Monday night compared to Tuesday night?
6. The six three-digit numbers are 123, 132, 231, 213, 312, 321. Their sum is 1332.
a. $1332 \div 37 = 36$
b. The sum is 3774. $3774 \div 37 = 102$
c. Yes. Each of the three columns of digits in the three-digit number will add up to the same number n . The sum will be $n(100 + 10 + 1)$ or $n(111)$. Since 111 is evenly divisible by 37, $111n$ is evenly divisible by 37.
7. Neither is cheaper since both approaches yield the same total purchase price. To determine this, let p represent any purchase price.
a. Discounted price: $p - p(d/100) = p(1 - d/100)$
Tax on discounted price: $p(1 - d/100)(t/100)$
Adding the two and simplifying: $p(1 - d/100) + p(1 - d/100)(t/100) = p(1 - d/100)(1 + t/100)$
b. Full price with tax: $p + p(t/100) = p(1 + t/100)$
Discount on full price with tax: $p(1 + t/100)(d/100)$
Subtracting the discount from the full price and simplifying: $p(1 + t/100) - p(1 + t/100)(d/100) = p(1 + t/100)(1 - d/100)$
These are the same since $a*b = b*a$.
8. Since $2/3 = 4/6$ and $1/2 = 3/6$, for every 4 gallons of blue paint you need 3 gallons of red paint to make the right purple mixture. Putting these amounts of paint together will generate 7 gallons of purple paint. Mixing 12 of these 7-gallon batches of paint will make the 84 gallons desired, so you will need $12*4 = 48$ gallons of blue paint and $12*3$ gallons of red paint
9. Each bicycle has 2 wheels and each tricycle has 3 wheels, and both have 2 pedals. For each tricycle, there is 1 more wheel than pedal. There are $176 - 152 = 24$ extra wheels, so there are 24 tricycles.
10. One possible answer: You are driving at 50 miles per hour to a city 240 miles away in two periods of driving. Let x represent the number of miles traveled in the first period. How many hours will it take to complete the trip?
11. The two paths are the same length. Let the distance from point A to point D be represented by t . The length of the first path (half the circumference of the circle with diameter d is $t = 1/2 \pi d$. The length of the second path (three halves of circles, one with diameter a , one with diameter b , and one with diameter c):
 $1/2(\pi a) + 1/2(\pi b) + 1/2(\pi c) = 1/2\pi (a+b+c) = 1/2 \pi d$.

Section II: Selected Response

12. d)

In partitive division a group of objects is partitioned into groups. In this case, the group of cookies is partitioned into bags (I) and the group of books is partitioned into boxes (III).

13. a. 3105/1000

b. d)

Since $\sqrt{2}$ is irrational, any number to which it is added will be irrational as well.

$\sqrt{2} < 2$, therefore $\sqrt{2}/1000 < .002$

$3.1 < 3.1 + \sqrt{2}/1000 < 3.11$

14. a. d)

Both 78 (I) and 84 (III) round to 80.

The sums which will round to 80 can be written in a systematic way allowing all of the questions to be answered by inspection. A portion of the full table follows:

| | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|
| 49 | 30 | 31 | 32 | 33 | 34 | 35 | | | | |
| 48 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | | | |
| 47 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | | |
| 46 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | |
| 45 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 44 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | |
| 43 | | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | |
| 42 | | | 33 | 34 | 35 | 36 | 37 | 38 | 39 | |
| 41 | | | | 34 | 35 | 36 | 37 | 38 | 39 | |
| 40 | | | | | 35 | 36 | 37 | 38 | 39 | |

b. 0.75

There would be 100 items if all were written, and there are 10 missing in the upper right-hand corner and 15 missing in the lower left-hand corner, so there are 75 out of 100 which round to 80 after being added. Thus the probability is $75/100 = 0.75$.

c. 0.15

There are 15 out of 100 which round to 70 after being added. Thus the probability is $15/100 = 0.15$.

d. 0.60

If both numbers round down, they are in the lower left-hand quarter of the table, which has 25 items. The 15 missing ones round to 70 when added. Thus the probability is $15/25 = 0.60$.

e. 0.40

If both numbers round down, they are in the lower left-hand quarter of the table, which has 25 items. Ten items round to 80 when added, so 10 out of 25 are the chances: $10/25 = 0.40$.

f. 0.40

If both numbers round up, they lie in the upper right-hand quarter, which is treated the same way: 10 out of 25 round to 90 when added.

g. 0.60

If both numbers round up, they lie in the upper right-hand quarter, which is treated the same way as in d. (above): 15 out of 25 round to 80 when added.

h. 1.00

The lower right-hand quarter contains the numbers in question, and all of them round to 80 when added.

15. a. b)

For any number to be divisible by 3, the sum of its digits must be divisible by 3.

b. e)

For any number to be divisible by 4, the last two digits must form a number divisible by 4. Therefore bc must be divisible by 4.

c. There are no possible answers. In order for a number to be divisible by 12 but not by 9, it must be divisible by 3 and by 4, but the sum of its digits must not be divisible by 9.

d. If $b=4$, $a=0$, 3 or 9

16. 52

Let b represent the number of bicycles in the store and t represent the number of tricycles. Equation A can be developed using the number of wheels: $2b+3t=176$. Equation B can be developed using the number of pedals: $2b+2t=152$. Subtracting equation B from A: $1t=24$. Substituting this value for t into equation B and solving for b , $b=52$.

17. 108

The combined rate of the two mills is $(150\text{lb.}/18\text{ min.}) + (100\text{lb.}/18\text{ min.}) = 250\text{lb.}/18\text{ min.}$ Since $1500 = 6 \cdot 250$, it will take six times as long, or $6 \cdot 18 = 108$ minutes to grind 1500 pounds.

18. d)

$$3^{20} = (3^2)^{10} = 9^{10}$$

$$2^{40} = (2^4)^{10} = 16^{10}$$

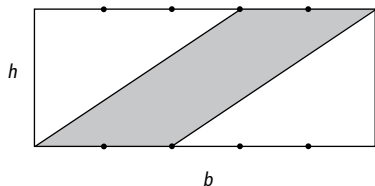
$$9^{10} < 10^{10} < 16^{10}$$

19. 5.00

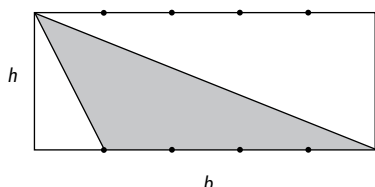
Initially the watermelon contained 9.9 pounds of water and 0.1 pounds of solids. After a while there will be less water, but still 0.1 pounds of solids, which represent 2% of the watermelon's total weight after it is brought up from the basement: $0.1 = 0.02x$, where x represents the watermelon's weight: $x = 5$.

20. a. c)

$$\text{Area of parallelogram: } A_1 = 2/5 b \cdot h$$

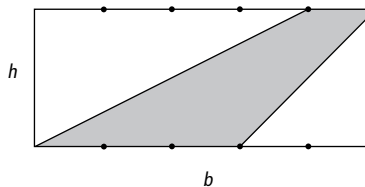


$$\text{Area of triangle: } A_2 = 1/2(4/5b) \cdot h = 2/5 b \cdot h$$

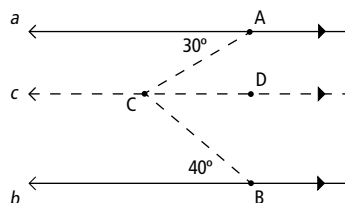


b. c)

$$\text{Area of trapezoid: } A_3 = 1/2h(3/5b + 1/5b) = 1/2h \cdot 4/5b = 2/5 b \cdot h$$



21. 70.00



Different approaches are possible, but one approach is to draw an auxiliary line¹ parallel to lines a and b through point C and add point D to line c :

$m \angle ACD = 30^\circ$ (This is an alternate interior angle to the acute angle with vertex A on line a .)

$m \angle DCB = 40^\circ$ (This is an alternate interior angle to the acute angle with vertex B on line b .)

$$m \angle ACD + m \angle DCB = m \angle ACB = 30^\circ + 40^\circ = 70^\circ$$

22. a. 6:1

If Asia has $3/5$ th of the world's population and $1/5$ th of the world's area, and the rest of the world has $2/5$ th of the world's population and $4/5$ th of its area, then the ratio of population density of the two is $3:1/2$ or $6:1$.

b. 330

If an average square mile of the world is placed in China and its size and population are changed to correspond to the relevant Chinese proportions of each relative to the world's, its size would shrink to $1/5$ th of a square mile (because China has 20% of the world's area), but its population would still be 60% of 110, or 66 people. Sixty-six people per $1/5$ th square mile equals 330 people per square mile.

¹ The function of auxiliary lines is to change difficult problems to simpler ones, often ones which have already been solved. Auxiliary lines could also be drawn perpendicular to line a through point A , creating a quadrilateral whose angles include $\angle ACB$ and can be solved, or perpendicular to line c through point C , creating two triangles, the solution of whose angles resolves the measurement of $\angle ACB$. An auxiliary line can also be drawn through points B and C ; its intersection with line a creates a triangle, the solution of whose angles resolves the measurement of $\angle ACB$.

c. 55

If the ratio of Asia's population density to the world's is 6:1, and China's population density is 330 people per square mile, the density of the world's population must be 330 people per square mile/6 = 55 people per square mile.

23. 567/990

Subtracting the decimal 0.572(m) from 57.27272 or 100 times the decimal (100 m) produces the equation $99m = 56.7$. Thus $m = 56.7/99$ or $567/990$.

24. a. 0.22

Area of the square hole: $(2r)^2 = 4r^2$
 Area of the round peg: πr^2
 Ratio of the open space to the total area of the hole: $(4r^2 - \pi r^2)/4r^2 = 1 - \pi/4 = 1 - 3.14/4 \approx 0.22$

b. 0.36

Area of round hole: πr^2
 Area of square peg: $(r\sqrt{2})^2 = 2r^2$
 Ratio of the open space to the total area of the hole: $(\pi r^2 - 2r^2)/\pi r^2 = 1 - 2/\pi = 1 - 2/3.14 \approx 0.36$

25. 180.00

Five triangles can be created in the star, each with two vertices at star points and one vertex as the interior angle of the convex pentagon with vertices $a, b, c, d,$ and e . The sum of the angles in these five triangles is $2(m\angle A + m\angle B + m\angle C + m\angle D + m\angle E) + (m\angle a + m\angle b + m\angle c + m\angle d + m\angle e) = 5 \cdot 180^\circ = 900^\circ$. Notice that each of the vertices at the star's points is counted twice in this process and the 540° of the pentagon's angles ($3 \cdot 180^\circ$) are also included: Solving for the sum of the star's angles only, $(m\angle A + m\angle B + m\angle C + m\angle D + m\angle E)$, yields 180° .

26. Setting up a proportion from similar triangles:

$2/22 = 1/11 = 6/x$, where x represents the distance to the person in feet: $x = 66$ feet.

27. a. 4

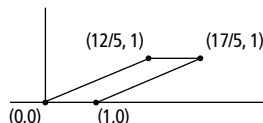
There are two symmetries that are reflections in lines, reflecting through one of the two lines joining opposite vertices, and there are two symmetries that are reflections in lines through the perpendicular bisectors of two opposite sides.

b. 4

There are four symmetries that are rotations through $90^\circ, 180^\circ, 270^\circ$ and 360° (the last of which is the same as not rotating at all).

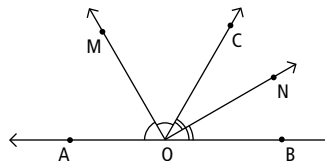
c. b)

28. Coordinates of two vertices: $x = 12/5, y = 1$;
 $x = 17/5, y = 1$



The parallelogram has a base of 1 unit, a height of 1 unit and two sides that are not bases of $13/5$ units. It can be drawn by placing one vertex at $(0,0)$ and another at $(1,0)$. The length of the two non-base sides are $13/5$ units, and these form the hypotenuses of two right triangles, each with one side equal to the parallelogram's height of 1 unit. By the Pythagorean theorem, the remaining side of the right triangles is $12/5$ units in length. Thus the third vertex must have a vertex at $(12/5, 1)$ and the fourth must project horizontally for 1 unit beyond that vertex to have coordinates of $(17/5, 1)$.

29. 090.00



$2m \angle MOC + 2m \angle CON = 180^\circ$
 Dividing all terms by 2 yields $m \angle MOC + m \angle CON = 90^\circ$
 $m \angle MOC + m \angle CON = m \angle MON = 90^\circ$

30. 16

Volume of smaller container: $V_1 = h\pi r^2 = 2$
 Volume of larger container: $V_2 = 2h\pi(2r)^2 = 8h\pi r^2 = 8V_1 = 16$

31. 6/36

1/36
 There are six ways of rolling a 7: $(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$ and 36 different ways of rolling any two numbers. Hence the probability of rolling a 7 is $6/36$. There are five ways of rolling a 6: $(1,5), (2,4), (3,3), (4,2), (5,1)$. Hence the probability of rolling a 6 is $5/36$.

Answer Sheet for Section II

12. a b c d e

13. a. [1]: 0 1 2 3 4 5 6 7 8 9 NA
 [2]: 0 1 2 3 4 5 6 7 8 9 NA
 [3]: 0 1 2 3 4 5 6 7 8 9 NA
 [4]: 0 1 2 3 4 5 6 7 8 9 NA
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 [6]: 0 1 2 3 4 5 6 7 8 9 NA
 [7]: 0 1 2 3 4 5 6 7 8 9 NA
 [8]: 0 1 2 3 4 5 6 7 8 9 NA

b. a b c d e

14. a. a b c d e

- b. [9]: 0 1 2 3 4 5 6 7 8 9 NA
 [10]: 0 1 2 3 4 5 6 7 8 9 NA
 [11]: 0 1 2 3 4 5 6 7 8 9 NA
- c. [12]: 0 1 2 3 4 5 6 7 8 9 NA
 [13]: 0 1 2 3 4 5 6 7 8 9 NA
 [14]: 0 1 2 3 4 5 6 7 8 9 NA
- d. [15]: 0 1 2 3 4 5 6 7 8 9 NA
 [16]: 0 1 2 3 4 5 6 7 8 9 NA
 [17]: 0 1 2 3 4 5 6 7 8 9 NA
- e. [18]: 0 1 2 3 4 5 6 7 8 9 NA
 [19]: 0 1 2 3 4 5 6 7 8 9 NA
 [20]: 0 1 2 3 4 5 6 7 8 9 NA
- f. [21]: 0 1 2 3 4 5 6 7 8 9 NA
 [22]: 0 1 2 3 4 5 6 7 8 9 NA
 [23]: 0 1 2 3 4 5 6 7 8 9 NA
- g. [24]: 0 1 2 3 4 5 6 7 8 9 NA
 [25]: 0 1 2 3 4 5 6 7 8 9 NA
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- h. [27]: 0 1 2 3 4 5 6 7 8 9 NA
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15. a. a b c d e

b. a b c d e

- c. [30]: 0 1 2 3 4 5 6 7 8 9 NA
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- d. [33]: 0 1 2 3 4 5 6 7 8 9 NA
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16. [36]: 0 1 2 3 4 5 6 7 8 9 NA
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17. [38]: 0 1 2 3 4 5 6 7 8 9 NA
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 [40]: 0 1 2 3 4 5 6 7 8 9 NA

18. a b c d e

19. [41]: 0 1 2 3 4 5 6 7 8 9 NA
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20. a. a b c d e

b. a b c d e

21. [44]: 0 1 2 3 4 5 6 7 8 9 NA
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- b. [50]: 0 1 2 3 4 5 6 7 8 9 NA
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- c. [53]: 0 1 2 3 4 5 6 7 8 9 NA
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- b. [64]: 0 1 2 3 4 5 6 7 8 9 NA
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- 25. [67]: 0 1 2 3 4 5 6 7 8 9 NA
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- 26. [72]: 0 1 2 3 4 5 6 7 8 9 NA
- [73]: 0 1 2 3 4 5 6 7 8 9 NA
- 27. a. [74]: 0 1 2 3 4 5 6 7 8 9 NA
- b. [75]: 0 1 2 3 4 5 6 7 8 9 NA
- c. a b c d e
- 28. a. [76]: 0 1 2 3 4 5 6 7 8 9 NA
- [77]: 0 1 2 3 4 5 6 7 8 9 NA
- [78]: 0 1 2 3 4 5 6 7 8 9 NA
- [79]: 0 1 2 3 4 5 6 7 8 9 NA
- b. [80]: 0 1 2 3 4 5 6 7 8 9 NA
- [81]: 0 1 2 3 4 5 6 7 8 9 NA
- [82]: 0 1 2 3 4 5 6 7 8 9 NA
- [83]: 0 1 2 3 4 5 6 7 8 9 NA
- 29. [84]: 0 1 2 3 4 5 6 7 8 9 NA
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- [98]: 0 1 2 3 4 5 6 7 8 9 NA